Subject: تکلیف ۱۹
Date

یلدا ترکی — ۲۰

۱-

$3x^2 - ax + 4 = 0 \qquad S = \alpha + 3\alpha = 4\alpha = \frac{a}{3}$

$P = \alpha \times 3\alpha = 3\alpha^2 = \frac{4}{3} \Rightarrow \alpha = \pm\frac{2}{3}$ ۵

$4 \times \frac{2}{3} = \frac{a}{3} \longrightarrow a = 8 \longrightarrow 8 - (-8) = 16$

$4 \times \frac{-2}{3} = \frac{a}{3} \longrightarrow a = -8$

۲-

$36x^2 - (m+14)x + 1 = 0 \qquad mx^2 + 2x + 2 = 0$

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \frac{\sqrt{\beta} + \sqrt{\alpha}}{\sqrt{\alpha\beta} \sim \frac{1}{6}} = 5 \Rightarrow \sqrt{\alpha} + \sqrt{\beta} = \frac{5}{6} \Rightarrow \alpha + \beta + 2\sqrt{\alpha\beta} = \frac{25}{36}$ ۵

$\frac{m+14}{36} + \frac{2}{6} = \frac{25}{36} \Rightarrow m + 14 + 12 = 25 \Rightarrow m = -1 \qquad \frac{2}{m} = \frac{2}{-1} = -2$

۳-

$4x^3 + kx^2 - 9x - 2 = 0 \quad , \quad \alpha\beta = -2 \longrightarrow \beta = \frac{-2}{\alpha}$

$\alpha + \beta = -1 \longrightarrow \alpha + \frac{-2}{\alpha} = 1 \Rightarrow \alpha^2 - \alpha - 2 = 0 \longrightarrow \alpha = 2 \ , \ \beta = -1 \longrightarrow x = 2$ ۵

$32 + 4k - 18 - 2 = 0 \Rightarrow k = -3$

۴-

$\alpha + \beta = -6 \qquad \alpha\beta = a$

$\alpha < \beta < 0 \longrightarrow 3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85 \longrightarrow \frac{5}{2}(\alpha^2 + \beta^2) + \frac{1}{2}(\alpha^2 - \beta^2)$

$\Rightarrow \alpha^2 + \beta^2 = 36 - 2\alpha\beta \qquad \alpha^2 - \beta^2 = -6\sqrt{\Delta}$ ۵

$12\sqrt{2} + 85 = \frac{5}{2}(36 - 2\alpha\beta) + \frac{1}{2}(-6\sqrt{\Delta}) \Rightarrow 24\sqrt{2} = -6\sqrt{\Delta} \Rightarrow \sqrt{\Delta} = -4\sqrt{2}$

$85 - 90 = -5\alpha\beta \longrightarrow \alpha\beta = 1 \Rightarrow P = 1 = \frac{a}{1} \Rightarrow \underline{a = 1}$

۵-

$x^4 - 7x^2 - 5 = 0 \Rightarrow t = x^2 \Rightarrow t^2 - 7t - 5 = 0 \xrightarrow{t>0} \frac{7 + \sqrt{49+20}}{2} \Rightarrow t = x^2 = \frac{7 + \sqrt{69}}{2}$

$\Rightarrow x = \pm\sqrt{\frac{7 + \sqrt{69}}{2}} \Rightarrow S = 0 \ , \ P = \frac{-7 - \sqrt{69}}{2} \longrightarrow 2P^2 - 3SP + 2S = 2P^2$ ۵

$= \frac{118 + 14\sqrt{69}}{2} = 59 + 7\sqrt{69}$

P4PCO

Subject:
Date

-۶

$2x^2 - ax + b = 0 \Rightarrow S = \frac{a}{2} \ , \ P = \frac{b}{2}$

$2ax^2 + ax - 6 \rightarrow S = -\frac{1}{2} \rightarrow \alpha + \beta = \frac{1}{2} = \frac{a}{2} \Rightarrow a = 1$

$\quad \llcorner P = -3 \rightarrow b = -6$

$\left[\frac{ab}{4}\right] = -2$

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$ax^2 - ax - b = 0 \qquad 40\beta^2 + 20\alpha^2 - 20\alpha = 17$

$\alpha = 1 - \beta \Rightarrow 40\beta^2 + 20(1-\beta)^2 - 20\beta = 7 \Rightarrow 20\beta(\underbrace{\beta - 1}_{-\alpha}) = -1 \Rightarrow \alpha\beta = \frac{-b}{a} = \frac{1}{20}$

$\Rightarrow a = -20b \qquad ax^2 - ax - b = 0 \Rightarrow -20bx^2 + 20bx - b = 0$

$\Rightarrow |\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{4}{\sqrt{5}}$

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$S = a + 1 = 2\alpha + 2$

$P = a = \alpha^2 + 2\alpha$

$\alpha^2 + 2\alpha + 1 = 2\alpha + 2 \rightarrow \alpha = \pm 1 \xrightarrow{\in N} \alpha = 1 \ , \ a = 3$

$S = 10 = 2\beta + 2 \rightarrow \beta = 4 \ , \ b = 6 \times 4 = 24 \qquad \Rightarrow b - a = 24 - 3 = 21$

-۹

$\alpha + \beta = \frac{-b}{a} = -1$

$\alpha\beta = -1 - m^2$

$\rightarrow \alpha^2 + \beta^2 = 1 - 2(-1 - m^2) = 2m^2 + 3 \xrightarrow{m^2 \geq 0} min = 3$

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رأس سهمی $A, B \rightarrow x_s = \frac{3-5}{2} = -1 \qquad y_s = 1$

$y - 1 = a(x+1)^2 \Rightarrow -1 = a(x+1)^2 \Rightarrow (x+1)^2 = \frac{-1}{a} \Rightarrow r = \sqrt{\frac{-1}{a}} \qquad x = -1 \pm r$

$\alpha = -1 + r \qquad \beta = -1 - r \Rightarrow (\alpha^2 + \beta^2) = 5 \rightarrow (-1+r)^2 + (-1-r)^2 = 5$

$\Rightarrow 2 + 2r^2 = 5 \rightarrow r^2 = \frac{3}{2}$

$\frac{-1}{a} = \frac{3}{2} \rightarrow a = \frac{-2}{3}$

$x = 0 \rightarrow y - 1 = a(1)^2 = a \Rightarrow y = a + 1 = \frac{-2}{3} + 1 = \frac{1}{3} \leftarrow$ عرض از مبدأ

PAPCO