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نام و نام خانوادگی [illegible] پاسخنامه تشریحی تکلیف شماره ۱۹ کلاس [illegible]

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$P=\frac{c}{a}=\frac{4}{3} / S=\frac{a}{3}=-\frac{b}{a}=\frac{a}{3}$

$K\times 3K=\frac{4}{3} \Rightarrow K^2=\frac{4}{9} \Rightarrow K=\frac{2}{3}$

$K+3K=\frac{a}{3} \Rightarrow K=\frac{a}{12} \Rightarrow \boxed{a=8}$

$3x^2-8x+4=0$

$\Delta=64-48=16$

$\frac{8\pm 4}{6} \begin{cases} +2 \\ \frac{4}{6}=\frac{2}{3} \end{cases}$

$3\alpha^2=\frac{4}{3} \to \alpha=\pm\frac{2}{3} \begin{cases} \alpha=\frac{2}{3} \to \beta=2 \to a=8 \\ \alpha=-\frac{2}{3} \to \beta=-2 \to a=-8 \end{cases}$ $\quad 8-(-8)=16$

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$\left(\frac{1}{\sqrt{x_1}}+\frac{1}{\sqrt{x_2}}=5\right)^2=\frac{1}{x_1}+\frac{1}{x_2}+\frac{2}{\sqrt{x_1x_2}}=25 \Rightarrow \left(\frac{x_1+x_2}{x_1x_2}+\frac{2}{\sqrt{x_1x_2}}=25\right)^{\times x_1x_2}$

$\Rightarrow x_1+x_2+2\sqrt{x_1x_2}=25x_1x_2 \Rightarrow \frac{m+14}{36}+2\sqrt{\cancel{\frac{1}{36}}}\frac{1}{6}=25\times\frac{1}{36} \Rightarrow \frac{m+14}{36}+\frac{\cancel{2}^{12}}{\cancel{6}_{36}}=\frac{25}{36}$

$\Rightarrow \frac{m+26}{36}=\frac{25}{36} \Rightarrow \boxed{m=-1}$ $\quad mx^2+3x+2 \to -x^2+3x+2=0$

حاصل ضرب ریشه‌ها $P=\frac{c}{a}=\boxed{-2}$

$S=x_1+x_2=-\frac{b}{a}=\frac{m+14}{36}$ $\quad P=x_1x_2=\frac{c}{a}=\frac{1}{36}$

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$4x^3+Kx^2-9x-2=0 \Rightarrow (x^2-x-2)(4x+1)$

$\alpha+\beta=1 \,,\, \alpha\beta=-2 \,,\, K=?$

$(x-\alpha)(x-\beta)=x^2-(\cancel{\alpha+\beta})^{1}x+\cancel{\alpha\beta}^{-2}=x^2-x-2$

$$
\begin{array}{l|l}
4x^3+Kx^2-9x+2 & x^2-x-2 \\
\hline
-4x^3 \mp 4x^2 \mp 8x & 4x+1 \\
Kx^2+4x^2-x-2 & \\
-x^2 \mp x \mp 2 & \\
Kx^2+3x^2=0 \Rightarrow \boxed{K=-3} &
\end{array}
$$

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$x^2+6x+a=0 \quad \alpha<\beta<0 \quad 3\alpha^2+2\beta^2=12\sqrt{2}+85 \quad a=?=\boxed{1}$

$\Delta=36-4a \Rightarrow \alpha,\beta=\frac{-6\pm\sqrt{\overset{4(9-a)}{36-4a}}}{2}=-3\pm\sqrt{9-a} \overset{\alpha<\beta}{\Rightarrow} \alpha=-3-\sqrt{9-a} \,,\, \beta=-3+\sqrt{9-a}$

$\alpha+\beta=-6$

$\alpha\beta=a \quad \alpha^2=18-a+6\sqrt{9-a} \,,\, \beta^2=18-a-6\sqrt{9-a}$

$3(18-a+6\sqrt{9-a})+2(18-a-6\sqrt{9-a})=85+12\sqrt{2} \Rightarrow 90-5a+\underbrace{6\sqrt{9-a}}=85+\underbrace{12\sqrt{2}}$

$6\sqrt{9-\alpha}=12\sqrt{2} \Rightarrow \sqrt{9-a}=2\sqrt{2}=\sqrt{8} \Rightarrow \boxed{a=1}$

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$x^4-7x^2-5=0$

$x^2=t \Rightarrow t^2-7t-5=0 \quad \Delta=49+20=69 \quad t=\frac{7\pm\sqrt{69}}{2} \begin{cases} \frac{7+\sqrt{69}}{2} \checkmark \\ \frac{7-\sqrt{69}}{2} \Rightarrow \times \end{cases}$ ریشه حقیقی $t=x^2$ ، $t<0$ نمی‌دهد

$\Rightarrow x=\pm\sqrt{\frac{7+\sqrt{69}}{2}}$ $\quad \alpha+\beta=0 \Rightarrow S=0$ $\quad \alpha\beta=P=+\sqrt{t}\times-\sqrt{t}=-t=-\frac{7+\sqrt{69}}{2}$

$2P^2-3SP+2S \Rightarrow 2\left(\frac{7+\sqrt{69}}{2}\right)^2-3\times\overset{0}{\cancel{S}}\times P+2\overset{0}{\cancel{S}}=2\times\frac{49+14\sqrt{69}+69}{4}=\frac{118+14\sqrt{69}}{2}$

$2x^2-ax+b=0 \quad x_1 , x_2$ ۶

$S=\frac{a}{2} , P=\frac{b}{2}$

$2ax^2+ax-9=0 \quad y_1 , y_2$

$S=-\frac{a}{2a}=-\frac{1}{2} , P=\frac{-9}{2a}=-\frac{3}{a}$

$x_1=y_1+\frac{1}{2} \Rightarrow x_1+x_2=y_1+y_2+1$

$x_2=y_2+\frac{1}{2} \Rightarrow \frac{a}{2}=\cancel{-\frac{1}{2}}+1 \Rightarrow \boxed{a=1}$

$x_1x_2=\underbrace{(y_1+\frac{1}{2})(y_2+\frac{1}{2})}_{y_1y_2+\frac{1}{2}(y_1+y_2)+\frac{1}{4}}=-\frac{3}{\cancel{a}}+\cancel{\frac{1}{2}(-\frac{1}{2})}+\cancel{\frac{1}{4}}=\boxed{-3}$

$x_1x_2=\frac{b}{2}=-3 \Rightarrow \boxed{b=-6}$

$\left[\frac{ab}{4}\right]-\left[\frac{1\times -6}{4}\right]=\boxed{-2} \Leftarrow$

$ax^2-ax-b=0 \qquad S=-\frac{b}{a}=\frac{a}{a}=1 , P=\alpha\beta=\frac{c}{a}=-\frac{b}{a}$ ۷

$40\beta^2+20\alpha^2-20\beta=17 \overset{\div 20}{\Rightarrow} \alpha^2+2\beta^2-\beta=\frac{17}{20}$

$a\alpha^2-a\alpha-b=0 \Rightarrow a(\alpha^2-\alpha)=b \Rightarrow \alpha^2=\alpha+\frac{b}{a} , \beta^2=\beta+\frac{b}{a}$

$\alpha^2+2\beta^2-\beta=(\alpha+\frac{b}{a})+\underbrace{2(\beta+\frac{b}{a})}_{2\beta+\frac{2b}{a}}-\beta \Rightarrow \underbrace{\alpha+\beta}_{1}+\frac{3b}{a}=\frac{17}{20} \Rightarrow \frac{3b}{a}=-\frac{3}{20} \Rightarrow \frac{b}{a}=-\frac{1}{20}$

$a^2+4ab=a^2(1+\frac{4b}{a})$

$\alpha-\beta=\frac{\sqrt{\Delta}}{|a|}=\frac{\sqrt{a^2+4ab}}{|a|} \Rightarrow \frac{\sqrt{a^2(1+\frac{4b}{a})}}{|a|}=\frac{\cancel{|a|}\sqrt{1+\frac{4b}{a}}}{\cancel{|a|}}=\sqrt{1+\frac{4b}{a}} \overset{\frac{b}{a}=-\frac{1}{20}}{\Rightarrow} \sqrt{1-\frac{\cancel{4}}{\cancel{20}}}=\boxed{\frac{2}{\sqrt{5}}}$

$x^2-(a+1)x+a=0$ دو ریشه فرد متوالی طبیعی $\Rightarrow 2n-1 , 2n+1 \Rightarrow 1 , 3$ ۸

$4n-1=4n^2-1 \Rightarrow 4n^2-4n=0 \qquad S=(2n-1)+(2n+1)=4n \qquad S=-\frac{b}{a}=a+1=4n \Rightarrow a=4n-1$

$\Rightarrow 4n(n-1)=0 \Rightarrow \boxed{n=1} , \boxed{a=3} \qquad P=(2n-1)(2n+1)=4n^2-1 \qquad P=\frac{c}{a}=a=4n^2-1=\boxed{3}$

$x^2-(3a+1)x+b=0$ دو ریشه زوج متوالی $\Rightarrow 2m , 2m+2 \qquad \to P=b=\boxed{24}$

$a=3 \Rightarrow x^2-10x+b=0 \qquad P=(2m)(2m+2)=4m^2+4m=\frac{c}{a}=b$

$-\frac{b}{a}=S=4m+2=3a+1=10 \Rightarrow \boxed{m=2}$

اصلاح

حاصلضرب ریشه‌ها $=24-3=\boxed{21}$

$x^2+x-1-m^2=0 \Rightarrow x^2+x-(1+m^2)=0$ ۹

$S=\alpha+\beta=-\frac{b}{a}=-1 \qquad \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$P=\alpha\beta=\frac{c}{a}=-(1+m^2) \qquad \Rightarrow (-1)^2-\underbrace{2(\cancel{-(1+m^2)})}_{+2+2m^2}=3+2m^2$

$m^2\geq 0 \Rightarrow$ if $m=0 \Rightarrow$ کمترین مقدار $=\boxed{3}$

$A(3 , y)$ عرض برابر $\qquad x=\frac{3+(-5)}{2}=-1 \qquad$ عرض راس سهمی $=1=-\frac{\Delta}{4a}$ ۱۰

$B(-5 , y) \quad \alpha^2+\beta^2=5 \qquad (-1 , +1)=$ راس سهمی $\to x=0$

$y=a(x-b)^2+K \Rightarrow y=a(x+1)^2+1 \Rightarrow y=ax^2+2ax+a+1 \quad y=-\frac{2}{3}+1=\boxed{\frac{1}{3}}$

محل برخورد با محور x

$0=a(x+1)^2+1 \Rightarrow a(x+1)^2=-1 \Rightarrow (x+1)^2=-\frac{1}{a} \Rightarrow |x+1|=\sqrt{-\frac{1}{a}} \Rightarrow x+1=\pm\sqrt{-\frac{1}{a}}$

$(x+1)^2\geq 0 \Rightarrow a<0$

$\Rightarrow x=-1\pm\sqrt{-\frac{1}{a}} \Rightarrow (1+\sqrt{-\frac{1}{a}})^2+(1-\sqrt{-\frac{1}{a}})^2=5 \Rightarrow -\frac{2}{a}+\cancel{2}=\cancel{5}^{3} \Rightarrow \boxed{a=-\frac{2}{3}}$