نام و نام خانوادگی [illegible] پاسخنامه تشریحی تکلیف شماره ۱۹ کلاس یازدهم تجربی B

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$P = \frac{c}{a} = \frac{4}{3} \,/\, S = \frac{a}{3} = -\frac{b}{a} = \frac{a}{3}$

$k \times 3k = \frac{4}{3} \Rightarrow k^2 = \frac{4}{9} \Rightarrow k = \frac{2}{3}$

$k + 3k = \frac{a}{3} \Rightarrow k = \frac{a}{12} \Rightarrow a = 8$

$3x^2 - 8x + 4 = 0$

$\Delta = 64 - 48 = 16$

$\frac{8 \pm 4}{6} \begin{cases} 2 \\ \frac{4}{6} = \frac{2}{3} \end{cases}$

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$\left(\frac{1}{\sqrt{x_1}} + \frac{1}{\sqrt{x_2}} = 5\right)^2 = \frac{1}{x_1} + \frac{1}{x_2} + \frac{2}{\sqrt{x_1x_2}} = 25 \Rightarrow \left(\frac{x_1+x_2}{x_1x_2} + \frac{2}{\sqrt{x_1x_2}} = 25\right) \times x_1x_2$

$\Rightarrow x_1 + x_2 + 2\sqrt{x_1x_2} = 25x_1x_2 \Rightarrow \frac{m+14}{36} + 2\sqrt{\frac{1}{36}} = 25 \times \frac{1}{36} \Rightarrow \frac{m+14}{36} + \frac{12}{36} = \frac{25}{36}$

$\Rightarrow \frac{m+26}{36} = \frac{25}{36} \Rightarrow m = -1$

$mx^2 + 3x + 2 \rightarrow -x^2 + 3x + 2 = 0$

حاصل ضرب ریشه‌ها $P = \frac{c}{a} = -2$

$S = x_1 + x_2 = -\frac{b}{a} = \frac{m+14}{36} \qquad P = x_1x_2 = \frac{c}{a} = \frac{1}{36}$

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$4x^3 + kx^2 - 9x - 2 = 0 \Rightarrow (x^2 - x - 2)(4x + 1)$

$\alpha + \beta = 1 \,,\, \alpha\beta = -2 \,,\, k = ?$

$(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - x - 2$

$4x^3 + kx^2 - 9x + 2 \;\big|\; x^2 - x - 2$, quotient $4x + 1$

$-4x^3 \mp 4x^2 \mp 8x$

$kx^2 + 4x^2 - x - 2$

$-x^2 \mp x \mp 2$

$kx^2 + 3x^2 = 0 \Rightarrow k = -3$

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$x^2 + 6x + a = 0 \qquad \alpha < \beta < 0 \qquad 3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85 \qquad a = ? = 1$

$\Delta = 36 - 4a \Rightarrow \alpha, \beta = \frac{-6 \pm \sqrt{36 - 4a}}{2} = -3 \pm \sqrt{9 - a} \overset{\alpha<\beta}{\Rightarrow} \alpha = -3 - \sqrt{9-a} ,\, \beta = -3 + \sqrt{9-a}$

$\alpha + \beta = -6$

$\alpha\beta = a \qquad \alpha^2 = 18 - a + 6\sqrt{9-a} ,\, \beta^2 = 18 - a - 6\sqrt{9-a}$

$3(18 - a + 6\sqrt{9-a}) + 2(18 - a - 6\sqrt{9-a}) = 85 + 12\sqrt{2} \Rightarrow 90 - 5a + 6\sqrt{9-a} = 85 + 12\sqrt{2}$

$6\sqrt{9-a} = 12\sqrt{2} \Rightarrow \sqrt{9-a} = 2\sqrt{2} = \sqrt{8} \Rightarrow a = 1$

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$x^4 - 7x^2 - 5 = 0$

$x^2 = t \Rightarrow t^2 - 7t - 5 = 0 \qquad \Delta = 49 + 20 = 69 \qquad t = \frac{7 \pm \sqrt{69}}{2} \begin{cases} \frac{7+\sqrt{69}}{2} \checkmark \\ \frac{7-\sqrt{69}}{2} \Rightarrow \times \end{cases}$ ریشه حقیقی نمی‌دهد $t = x^2 ,\, t < 0$

$\Rightarrow x = \pm\sqrt{\frac{7+\sqrt{69}}{2}} \qquad \alpha + \beta = 0 \Rightarrow S = 0 \qquad \alpha\beta = P = +\sqrt{t} \times -\sqrt{t} = -t = -\frac{7+\sqrt{69}}{2}$

$2P^2 - 3SP + 2S \Rightarrow 2\left(\frac{7+\sqrt{69}}{2}\right)^2 - 3 \times 0 \times P + 2 \times 0 = 2 \times \frac{49 + 14\sqrt{69} + 69}{4} = \frac{118 + 14\sqrt{69}}{2}$

$2x^2-ax+b=0$ x_1, x_2 $S=\frac{a}{2}$, $P=\frac{b}{2}$

$2ax^2+ax-4=0$ y_1, y_2 $S=-\frac{a}{2a}=-\frac{1}{2}$, $P=\frac{-4}{2a}=-\frac{2}{a}$

$\left[\frac{ab}{2}\right]-\left[\frac{|x-4|}{2}\right]=-2$ <-

$x_1=y_1+\frac{1}{2}$, $x_2=y_2+\frac{1}{2}$ $\Rightarrow x_1+x_2=y_1+y_2+1$

$\frac{a}{2}=-\frac{1}{2}+1=\frac{1}{2} \Rightarrow a=1$

$x_1x_2=(y_1+\frac{1}{2})(y_2+\frac{1}{2})$

$y_1+y_2+\frac{1}{2}(y_1+y_2)+\frac{1}{4}=-\frac{2}{4}+\frac{1}{2}(-1)+\frac{1}{4}=-2$

$x_1x_2=\frac{b}{2}=-2 \Rightarrow b=-4$

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$ax^2-ax-b=0$

$S=-\frac{b}{a}=\frac{a}{a}=1$, $P=\alpha\beta=\frac{c}{a}=-\frac{b}{a}$

$40\beta^2+20\alpha^2-20\beta=17 \div 20 \Rightarrow \alpha^2+2\beta^2-\beta=\frac{17}{20}$

$a\alpha^2-a\alpha-b=0 \Rightarrow a(\alpha^2-\alpha)=b \Rightarrow \alpha^2=\alpha+\frac{b}{a}$, $\beta^2=\beta+\frac{b}{a}$

$\alpha^2+2\beta^2-\beta=(\alpha+\frac{b}{a})+2(\beta+\frac{b}{a})-\beta \Rightarrow \underbrace{\alpha+\beta}_{1}+\frac{2b}{a}=\frac{17}{20} \Rightarrow \frac{3b}{a}=-\frac{3}{20} \Rightarrow \frac{b}{a}=-\frac{1}{20}$

$a^2+2ab=a^2(1+\frac{2b}{a})$

$\alpha-\beta=\frac{\sqrt{5}}{|a|}=\frac{\sqrt{a^2-2ab}}{|a|} \Rightarrow \frac{\sqrt{a^2(1+\frac{2b}{a})}}{|a|}=\frac{|a|\sqrt{1+\frac{2b}{a}}}{|a|}=\sqrt{1+\frac{2b}{a}} \overset{\frac{b}{a}=-\frac{1}{20}}{=} \sqrt{1-\frac{2}{20}}=\frac{2}{\sqrt{5}}$

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$x^2-(a+1)x+a=0$ دوجمله‌ای متوالی طبیعی $\Rightarrow 2n-1, 2n+1 \Rightarrow 1, 3$

$4n-1=4n^2-1 \Rightarrow 4n^2-4n=0$

$\Rightarrow 4n(n-1)=0 \Rightarrow n=1$, $a=3$

$S=(2n-1)+(2n+1)=4n$ $S=-\frac{b}{a}=a+1=4n \Rightarrow a=4n-1$

$P=(2n-1)(2n+1)=4n^2-1$ $P=\frac{c}{a}=a=4n^2-1=3$

$x^2-(2a+1)x+b=0$ دوجمله‌ای متوالی $\Rightarrow 2m, 2m+2$

$a=3 \Rightarrow x^2-10x+b=0$

$P=(2m)(2m+2)=4m^2+4m=\frac{c}{a}=b \rightarrow P=b=24$

$-\frac{b}{a}=S=4m+2=2a+1=10 \Rightarrow m=2$

اختلاف $=24-3=21$

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$x^2+x-1-m^2=0 \Rightarrow x^2+x-(1+m^2)=0$

$S=\alpha+\beta=-\frac{b}{a}=-1$

$P=\alpha\beta=\frac{c}{a}=-(1+m^2)$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\Rightarrow (-1)^2-2(\underbrace{-(1+m^2)}_{-1-m^2})=3+2m^2$

$+2+2m^2$

$m^2 \neq 0 \Rightarrow$ if $m=0 \Rightarrow$ کمترین مقدار $=3$

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$A(3,y)$ عرض برابر

$B(-5,y)$ $\alpha^2+\beta^2=a$

$x=\frac{3+(-5)}{2}=-1$ عرض رأس سهمی $=1=-\frac{\Delta}{4a}$

رأس سهمی $(-1,+1)$ $\rightarrow x=0$

$y=a(x-b)^2+k \Rightarrow y=a(x+1)^2+1 \Rightarrow y=ax^2+2ax+a+1$ $y=-\frac{2}{3}+1=\frac{1}{3}$

$0=a(x+1)^2+1 \Rightarrow a(x+1)^2=-1 \Rightarrow (x+1)^2=-\frac{1}{a} \Rightarrow |x+1|=\sqrt{-\frac{1}{a}} \Rightarrow x+1=\pm\sqrt{-\frac{1}{a}}$ ← عرض‌های برابر x

$(x+1)^2>0 \Rightarrow a<0$

$\Rightarrow x=-1\pm\sqrt{-\frac{1}{a}} \Rightarrow (1+\sqrt{-\frac{1}{a}})^2+(1-\sqrt{-\frac{1}{a}})^2=5 \Rightarrow -\frac{2}{a}+2=5 \Rightarrow a=-\frac{2}{3}$

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