

باز هم رخت B

شماره تکلیف ۲۰

مسئله ساری ۲۰

برای دو عدد a و b که رابطه $2a^2 - a + 4 = 0$ برقرار می‌باشد. اختلاف این دو عدد را بیابید.

برای دو عدد a و b که رابطه $2a^2 - a + 4 = 0$ برقرار می‌باشد.

$$\alpha = 3\beta \Rightarrow 4\beta = \frac{9}{\beta} \Rightarrow \alpha = 3 \times \frac{1}{3} \Rightarrow \alpha = 1$$

$$3\beta \times \beta = 3\beta^2 \Rightarrow 3\beta^2 = \frac{9}{\beta} \Rightarrow \beta^2 = \frac{3}{\beta} \Rightarrow \beta^3 = 3 \Rightarrow \beta = \sqrt[3]{3}$$

$$2 + \frac{1}{\beta} = \frac{9}{\beta} \Rightarrow \frac{1}{\beta} = \frac{7}{\beta} \Rightarrow \alpha_1 = 1 \quad \alpha_1 - \alpha_2 = 1 - (-1) = 2$$

$$\beta = -\frac{1}{3} \Rightarrow \alpha = -2 \quad -2 - \frac{1}{3} = \frac{7}{3} \Rightarrow \alpha_2 = -1$$

تجمع چند سلسله ریاضی که داده شده است. حاصل آن را بیابید.

$$\sqrt{\frac{1}{a}} + \sqrt{\frac{1}{b}} = a \Rightarrow \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = a \Rightarrow \frac{\sqrt{b} + \sqrt{a}}{\sqrt{ab}} = a \Rightarrow \sqrt{a} + \sqrt{b} = \frac{a}{\sqrt{b}}$$

$$\alpha\beta = \frac{1}{39} \quad \alpha + \beta = \frac{m+14}{39} \quad \alpha + \beta + 2\sqrt{\frac{1}{39}} = \frac{25}{39} \Rightarrow \frac{m+14}{39} = \frac{25}{39} - \frac{12}{39} \Rightarrow \frac{m+14}{39} = \frac{13}{39} \Rightarrow m+14=13 \Rightarrow m=-1$$

$$m\alpha^2 + 3m + 2 = 0$$

$$-m^2 + 3m + 2 = 0 \Rightarrow \frac{C}{a} = -2$$

اگر α, β ریشه‌های معادله $kx^2 + kx^2 - 9x - 2 = 0$ باشد، $\alpha\beta = -2$ و $\alpha + \beta = 1$ را داریم.

$$\left. \begin{aligned} (\alpha + \beta)^2 &= 1 \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 1 \Rightarrow \alpha^2 + \beta^2 = 5 \\ \alpha + \beta &= 1 \end{aligned} \right\} \begin{aligned} \alpha &= 2 \\ \beta &= -1 \end{aligned}$$

$$kx(-1)^2 + k + 9 - 2 = 0 \Rightarrow k = -3$$

$$kx^2 + kx - 18 - 2 = 0 \Rightarrow k = -3$$

اگر α, β ریشه‌های معادله $x^2 + 4x + a = 0$ باشد، $\alpha^2 + 2\beta^2 = 12\sqrt{2} + 15$ و $\alpha < \beta < 0$ را داریم.

$$2\alpha^2 + 2\beta^2 = 12\sqrt{2} + 15 \Rightarrow \alpha^2 + \beta^2 = 6\sqrt{2} + \frac{15}{2}$$

$$\frac{a}{r}(\alpha^2 + \beta^2) + \frac{1}{r}(\alpha^2 - \beta^2) \Rightarrow \alpha^2 + \beta^2 = 5r - 2a \Rightarrow \frac{r}{2} - 2a = 6\sqrt{2} + \frac{15}{2} \Rightarrow \alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta) = -4(-\sqrt{49 - 4a}) = 4\sqrt{49 - 4a}$$

$$\frac{a}{r}(\alpha^2 + \beta^2) + \frac{1}{r}(\alpha^2 - \beta^2) \Rightarrow 4 - 5a + 3\sqrt{49 - 4a} = 12\sqrt{2} + 15$$

$$4 - 5a = 15 \Rightarrow a = 1 \quad 3\sqrt{49 - 4a} = 12\sqrt{2} \Rightarrow a = 1$$

اگر مجموع دو صوابی مرتب شده حاصل‌ضرب $2x^2 - 35x + 15 = 0$ باشد، p و q را بیابید.

$$x^2 = t \quad t = \frac{35 \pm \sqrt{1225 - 120}}{2} \quad 2x^2 - 35x + 15 = 0$$

$$t^2 - 35t + 15 = 0$$

$$\Delta = 1225 - 120 = 1105$$

$$x = \frac{35 \pm \sqrt{1105}}{2} \quad 2 = 0 \Rightarrow p = \frac{35 - \sqrt{1105}}{2}$$

$$2px - 35x + 15 = 0 \Rightarrow 2px = \frac{35 + 49 + 14\sqrt{1105}}{2} \times 2 = 59 + 7\sqrt{1105}$$

رہے گا سہ $2n^2 - an + b = 0$ ہم فرض کریں گے کہ $2an^2 + an - 4 = 0$ نسبت قرار $\left\{ \frac{ab}{c} \right\}$ ایسا۔

$$\begin{aligned} \alpha + \beta &= -\frac{1}{2} & \alpha + \frac{1}{\alpha} + \beta + \frac{1}{\beta} &= \frac{a}{2} \\ \alpha\beta &= -\frac{2}{a} & \alpha + \beta + 1 &= \frac{a}{2} \end{aligned}$$

6

$$\alpha + \beta = -\frac{1}{2}$$

$$\alpha + \beta = \frac{a-2}{2} \Rightarrow a = 1$$

$$\left(\frac{1}{2} + \alpha \right) \left(\frac{1}{2} + \beta \right) = \frac{1}{4} + (\alpha + \beta) \frac{1}{2} + \frac{\alpha\beta}{2} = \frac{b}{2}$$

$$\frac{1}{4} - \frac{1}{4} - \frac{1}{2} = \frac{b}{2} \Rightarrow b = -1$$

$$\left\{ \frac{ab}{c} \right\} = \left\{ \frac{1 \times -1}{2} \right\} = \left\{ -\frac{1}{2} \right\} = \left\{ -\frac{1}{2} \right\}$$

اگر α, β رہے گا تین $an^2 - an - b = 0$, $2\alpha^2 + 2\alpha - 2\beta = 17$, $2\beta^2 + 2\beta - 2\alpha = 17$ اختلافت رہے گا سہ ایسا۔

$$\begin{aligned} 2\alpha^2 - 2\alpha - b &= 0 & 2\beta^2 - 2\beta - b &= 0 \\ 2\alpha^2 - 2\alpha - b &= 0 & 2\beta^2 - 2\beta - b &= 0 \end{aligned}$$

7

$$2(\alpha^2 - \beta^2) + 2(\alpha - \beta) = 0 \Rightarrow 2(\alpha + \beta)(\alpha - \beta) + 2(\alpha - \beta) = 0$$

$$2(\alpha - \beta) \left(\alpha + \beta + 1 \right) = 0$$

$$\alpha + \beta + 1 = 0 \Rightarrow \alpha + \beta = -1$$

$$\frac{b}{2} = -\frac{1}{2} \Rightarrow b = -1$$

$$\frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{a^2 + 4ab}}{|a|} = \frac{\sqrt{a^2 - 4a}}{|a|} = \frac{\sqrt{a(a-4)}}{|a|} = \frac{\sqrt{a}}{\sqrt{a-4}}$$

رہے گا سہ $n^2 - (a+1)n + a = 0$ درجہ فرد نسبتیں ہیں، رہے گا سہ $n^2 - (a+1)n + a = 0$ $2k+1$ $2k-1$

$$\begin{aligned} a &= 2k+1 \times 2k-1 = 4k^2 - 1 \\ a+1 &= 4k \Rightarrow a = 4k-1 \end{aligned}$$

8

$$4k^2 - 4k = 0 \Rightarrow k(k-1) = 0 \Rightarrow k = 1, k = 0$$

$$a = 3$$

$$\begin{aligned} b &= 4k^2 + 4k & 2a+1 &= 4k^2 + 4k \\ 10 &= 4k^2 + 4k \Rightarrow k^2 + k - 5 = 0 & |a-b| &= |3-24| = 21 \end{aligned}$$

اگر α, β رہے گا سہ $n^2 + n - 1 - m^2 = 0$ نسبت قرار ملے گی ایسا۔

$$\begin{aligned} \alpha + \beta &= -1 \\ \alpha\beta &= -1 - m^2 & \alpha^2 + \beta^2 &= 8^2 - 2\alpha\beta = 1 - 2(-m^2 - 1) = 2 + 2m^2 \end{aligned}$$

$$m^2 = 0 \Rightarrow 2 + 2m^2 = 2 \Rightarrow \boxed{2}$$

9

نقطہ $A(3, 9)$, $B(-5, 9)$ دو نقطہ ہیں جن کے درمیان میں $n^2 + n - 1 - m^2 = 0$ نسبت قرار ملے گی ایسا۔

نسبت قرار ملے گی α, β رہے گا سہ $\alpha^2 + \beta^2 = 5$, $\alpha + \beta = 5$ ایسا۔

$$\begin{aligned} y &= a(n+1)^2 + 1 \Rightarrow 0 = a(n^2 + 2n + 1) + 1 \Rightarrow 0 = an^2 + 2an + a + 1 \\ \alpha + \beta &= -\frac{2a}{a} = -2 & \alpha\beta &= \frac{a+1}{a} & \alpha^2 + \beta^2 + 2\alpha\beta &= 4 \Rightarrow 2\alpha\beta = -4 \\ \alpha\beta &= -2 & \alpha\beta &= -\frac{4}{2} = -2 \end{aligned}$$

10

$$\frac{a+1}{a} = -\frac{1}{2} \Rightarrow a = -\frac{2}{3}$$

$$y = -\frac{2}{3}(n+1)^2 + 1 \Rightarrow y = -\frac{2}{3}n^2 - \frac{4}{3}n + \frac{1}{3}$$

$$n = 0 \Rightarrow y = \frac{1}{3}$$