۱

$\alpha = 3\beta$

$\alpha\beta = 3\beta^2 = \frac{4}{3} \Rightarrow \beta^2 = \frac{4}{9} \Rightarrow$

$\beta = \frac{2}{3} \Rightarrow \alpha = 3\times\frac{2}{3} = 2 \Rightarrow S = \frac{8}{3} = \frac{-a}{3} \Rightarrow \alpha = -1$

$\beta = -\frac{2}{3} \Rightarrow \alpha = -2 \Rightarrow S = \frac{-8}{3} = \frac{-a'}{3} \Rightarrow a' = 1$

$|a' - a| = \boxed{16}$

۲

$S = \frac{m+14}{36}$, $P = \frac{1}{36}$

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5$

$\Rightarrow mx^2 + 3x + 2 = -x^2 + 3x + 2 = 0 \Rightarrow P' = \boxed{-2}$

$\frac{\sqrt{S+2\sqrt{P}}}{\sqrt{P}} = \frac{\sqrt{\frac{m+14}{36} + 2\sqrt{\frac{1}{36}}}}{\sqrt{\frac{1}{36}}} = \frac{\frac{\sqrt{m+24}}{6}}{\frac{1}{6}} = 5 \Rightarrow m + 24 = 25 \Rightarrow m = -1$

۳

$\alpha + \beta = 1 \Rightarrow \alpha = 1 - \beta$

$\alpha\beta = -2$

$(1-\beta)\beta = -2$

$\beta - \beta^2 = -2$

$\beta^2 - \beta - 2 = 0$

$\beta = 2/-1 \Rightarrow 4(-1)^3 + K(-1)^2 - 2(-1) - 2 = K + 3 = 0 \Rightarrow K = \boxed{-3}$

۴

$S = -6$, $P = a$, $\Delta = 36 - 4a \Rightarrow \alpha = \frac{-\sqrt{36-4a} - 6}{2} = -\sqrt{9-a} - 3$, $\beta = \sqrt{9-a} - 3$

$\alpha^2 + \beta^2 = S^2 - 2P = 36 - 2a$

$3(-\sqrt{9-a} - 3)^2 + 2(\sqrt{9-a} - 3)^2 = 27 + 27 - 3a + 18\sqrt{9-a} + 18 - 2a + 18 + 12\sqrt{9-a}$

$= 90 - 5a + 6\sqrt{9-a} = 12\sqrt{2} + 18$

$5 - 5a + 6\sqrt{9-a} = 12\sqrt{2} \Rightarrow \boxed{a = 1}$

با امتحان کردن ریشه حدسی

۵

$t = x^2 \Rightarrow t^2 - 7t - 5 = 0 \Rightarrow \Delta = 49 + 20 = 69 \Rightarrow t_1 = \frac{\sqrt{69}+7}{2}$ قبول

$t_2 = \frac{-\sqrt{69}+7}{2}$ غ ق ق $(x^2 \geq 0)$

$x^2 = \frac{\sqrt{69}+7}{2} \Rightarrow \alpha = \sqrt{\frac{\sqrt{69}+7}{2}}$

$\beta = -\sqrt{\frac{\sqrt{69}+7}{2}}$

$\Rightarrow S = 0$, $P = -\frac{\sqrt{69}+7}{2}$

$2P^2 - \frac{3}{5}SP + \frac{2}{5}S = 2P^2 = 2\left(-\frac{\sqrt{69}+7}{2}\right)^2 = 8\times\frac{118+14\sqrt{69}}{16} = \boxed{59 + 7\sqrt{69}}$

$S-1=S'$

$\frac{a}{2}-1=\frac{-a}{2a}=\frac{-1}{2}$

$a-2=-1 \Rightarrow a=1$

$(\alpha+\frac{1}{2})(\beta+\frac{1}{2})=P'=\frac{-6}{2\times 1}=-3$

$\alpha\beta+\frac{1}{2}(\alpha+\beta)+\frac{1}{4}=-3$

$\Downarrow S=\frac{1}{2}, P=\frac{b}{2}$

$\frac{b}{2}+\frac{1}{2}\times\frac{1}{2}+\frac{1}{4}=-3$

$\frac{b}{2}+\frac{1}{2}=-3$

$b+1=-6 \Rightarrow b=-7$

$\left[\frac{ab}{4}\right]=\left[\frac{1\times(-7)}{4}\right]=\left[\frac{-7}{4}\right]=-2$

۶ (۱)

$2x^2-ax+b=0 \begin{cases} S=\frac{a}{2} \\ P=\frac{b}{2} \end{cases}$ $2ax^2+ax-4 \begin{cases} S=\frac{1}{2} \to \alpha+\beta=\frac{1}{2}=\frac{a}{2} \to a=1 \\ P=-2 \to b=-4 \end{cases}$

$\left[\frac{ab}{4}\right]=-2$

$\alpha=1-\beta$ $\quad 4\beta^2+2(1-\beta)^2-2\beta=7 \Rightarrow 2\beta(\underbrace{\beta-1}_{-\alpha})=-1 \to \alpha\beta=\frac{1}{2}=\frac{-b}{a}$

$\Rightarrow a=-2b$ $\quad ax^2-ax-b=0 \to -2bx^2+2bx-b=0$ $\quad |\alpha-\beta|=\frac{\sqrt{\Delta}}{|a|}=\frac{2}{\sqrt{5}}$

۷ (۲) (۷)

$S=\alpha+1 \Rightarrow$ ریشه فرد کوچکتر $=x$ $\quad x+x+2=\alpha+1 \Rightarrow 2x+1=\alpha=P \Rightarrow 2x+1=x(x+2)=x^2+2x$

$P=\alpha$ $\quad \alpha=2x+1=3$ $\quad x^2=1$

$\begin{cases} x_1=1 \text{ ق ق} \\ x_2=-1 \text{ غ ق ق} \end{cases}$ $(x\in N)$

$S'=3\alpha+1=10 \Rightarrow$ ریشه زوج کوچکتر $=y$ $\quad y+y+2=10$

$P'=b$ $\quad 2y=8 \Rightarrow y=4 \Rightarrow y+2=6$ $\quad x+2=3 \Rightarrow$ اول ریشه های معادله اول

$|P'-P|=|4\times 6-1\times 3|=21$ $\quad$ ل) ۶ و ۴ ریشه های معادله دوم

۸ (۵)

$\alpha^2+\beta^2=S^2-2P=1-2(-1-m^2)=3+2m^2$ $\quad \Rightarrow \alpha^2+\beta^2$ کمترین مقدار $=3+2(0)^2=3$

کمترین مقدار این عبارت زمانی هست که m^2 کمترین مقدار باشد. و کمترین مقدار m^2، صفر است.

$S=-1$

$P=-1-m^2$

۹ (۵)

$x_S=\frac{-\Delta+3}{2}=-1=\frac{-b}{2a} \Rightarrow b=2a$ $\quad \Rightarrow S=-2$, $P=\frac{a+1}{a}$

$y_S=1=\frac{4ac-b^2}{4a} \Rightarrow 4a=4ac-4a^2 \Rightarrow 1=c-a \Rightarrow c=a+1$

$\alpha^2+\beta^2=5 \Rightarrow S^2-2P=4-2(\frac{a+1}{a})=5 \Rightarrow \frac{-2a-2}{a}=1 \Rightarrow -2a-2=a$

$3a=-2$

$a=\frac{-2}{3}$

$c=a+1=-\frac{2}{3}+1=\frac{1}{3}$

۱۰ (۵)