Subject
Date

فاطمه متقی - ۱۹   به نام خدا

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۱)

$۳x^۲-ax+۴=۰ \qquad S=\alpha+۳\alpha=۴\alpha=\frac{a}{۳}$

$P=\alpha\cdot۳\alpha=۳\alpha^۲=\frac{۴}{۳}\to\alpha^۲=\frac{۴}{۹}\to\alpha=\pm\frac{۲}{۳}$

$\to ۴\times\frac{۲}{۳}=\frac{a}{۳}\Rightarrow a=۸$

$\to ۴\times\frac{-۲}{۳}=\frac{a}{۳}\Rightarrow a=-۸$

$\to ۸-(-۸)=\boxed{۱۶}$

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۲)

$۳۶x^۲-(m+۱۴)x+۱=۰ \qquad mx^۲+۳x+۲=۰$

$\frac{۱}{\sqrt{\alpha}}+\frac{۱}{\sqrt{\beta}}=\frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}}=۵ \xrightarrow{\text{توان ۲}} \frac{\alpha+\beta+۲\sqrt{\alpha\beta}}{\alpha\beta\,,۳۶}=۲۵ \to \alpha+\beta+۲\sqrt{\alpha\beta}=\frac{۲۵}{۳۶}$

$\frac{m+۱۴}{۳۶}+\frac{۲}{۶}=\frac{۲۵}{۳۶} \xrightarrow{\text{طرفین}} m+۱۴+۱۲=۲۵ \to m=-۱ \Rightarrow \frac{۲}{m}=\frac{۲}{-۱}=\boxed{-۲}$

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۳)

$۴x^۳+kx^۲-۹x-۲=۰$

$\alpha+\beta=-۱ \qquad \to \alpha+\frac{-۲}{\alpha}=۱ \to \alpha^۲-\alpha-۲=۰ \to (\alpha-۲)(\alpha+۱)$

$\alpha\beta=-۲ \to \beta=\frac{-۲}{\alpha} \qquad \alpha=۲,-۱$

$\begin{cases}\alpha=۲\\ \beta=-۱\end{cases} \xrightarrow{x=۲} ۳۲+۴k-۱۸-۲=۰ \Rightarrow \boxed{k=-۳}$

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۴)

$\alpha+\beta=-۶ \qquad \alpha\beta=a \qquad \alpha<\beta<۰$

$۳\alpha^۲+۲\beta^۲=۱۲\sqrt{۲}+۸۵ \to \frac{۵}{۲}(\alpha^۲+\beta^۲)+\frac{۱}{۲}(\alpha^۲-\beta^۲) \to \alpha^۲+\beta^۲=۳۶-۲\alpha\beta$

$\alpha^۲-\beta^۲=S\cdot P=-۶\sqrt{\Delta} \Longrightarrow ۱۲\sqrt{۲}+۸۵=\frac{۵}{۲}(۳۶-۲\alpha\beta)+\frac{۱}{۲}(-۶\sqrt{\Delta})$

$\to ۲۴\sqrt{۲}=-۶\sqrt{\Delta} \to \sqrt{\Delta}=-۴\sqrt{۲} \qquad ۳۴-۳۶=-۲\alpha\beta \to \alpha\beta=۱ \to \boxed{P=۱=a}$

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۵)

$x^۴-۷x^۲-۵=۰ \to t=x^۲ \to t^۲-۷t+۵=۰ \xrightarrow{t>۰} \frac{۷+\sqrt{۴۹+۲۰}}{۲}$

$\to t=\frac{۷+\sqrt{۶۹}}{۲}=x^۲ \to x=\pm\sqrt{\frac{۷+\sqrt{۶۹}}{۲}}$

$\to S=۰ \ , \ P=\frac{-۷-\sqrt{۶۹}}{۲} \to ۲P^۲-۳SP+۲S=۲P^۲=\frac{۱۱۸+۱۴\sqrt{۶۹}}{۲}=۵۹+۷\sqrt{۶۹}$

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۶)

$۲x^۲-ax+b=۰ \to S=\frac{a}{۲} \ , \ P=\frac{b}{۲}$

$۲ax^۲+ax-۶=۰ \to S=\frac{-۱}{۲} \ , \ P=-۳$

$\to \begin{matrix}a=۱\\ b=-۶\end{matrix} \Rightarrow \left[\frac{ab}{۴}\right]=\left[\frac{-۶}{۴}\right]=-۲$

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$a x^2 - a x - b = 0 \to S = 1 \rightsquigarrow \alpha = 1 - \beta$

$40\beta^2 + 20\underbrace{\alpha^2}_{(1-\beta)^2} - 20\beta = 17 \Rightarrow 20\beta\underbrace{(\beta - 1)}_{-\alpha} = -1 \Rightarrow \alpha\beta = \frac{1}{20} = \frac{-b}{a} \to a = -20b$

$\hookrightarrow -20b x^2 + 20b x - b = 0 \Rightarrow |\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{320 b^2}}{|-20b|} = \sqrt{\frac{8}{10}} = \boxed{\frac{2}{\sqrt{5}}}$

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$S = a + 1 \to 2\alpha + 2$

$P = a \to \alpha^2 + 2\alpha \longrightarrow \alpha^2 + 2\alpha + 1 = 2\alpha + 2 \Rightarrow \alpha = \pm 1 \Rightarrow \alpha = 1 \rightsquigarrow a = 3$

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$S = 10 = 2\beta + 2 \to \beta = 4 \qquad b = 6 \times 4 = 24 \longrightarrow b - a = 24 - 3 = \boxed{21}$

(۹

$\alpha + \beta = \frac{-b}{a} = -1$

$\alpha\beta = -1 - m^2 \to \alpha^2 + \beta^2 = S^2 - 2P = 1 - 2(-1 - m^2) = 2m^2 + 3$

$m^2 \geq 0 \hookrightarrow \boxed{\min = 3}$

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$A\left|\begin{matrix}3\\ y\end{matrix}\right. \quad B\left|\begin{matrix}-5\\ y\end{matrix}\right. \to$ عرض یکسان $\Rightarrow x_{\text{رأس}} = \frac{-5+3}{2} = -1 \Rightarrow S_{\text{رأس}}\left|\begin{matrix}-1\\ 1\end{matrix}\right.$

$\hookrightarrow y - 1 = a(x+1)^2 \Rightarrow \frac{-1}{a} = (x+1)^2 \xrightarrow{t = \sqrt{\frac{-1}{a}}} x = -1 \pm t \to \alpha = -1 + t,\ \beta = -1 - t$

$\alpha^2 + \beta^2 = 5 \to (-1+t)^2 + (-1-t)^2 = 2 + 2t^2 \Rightarrow 2t^2 + 2 = 5$

$t^2 = \frac{3}{2} \to \frac{-1}{a} = \frac{3}{2} \to a = \frac{-2}{3}$

$\xrightarrow[\text{در معادله}]{x=0} y - 1 = a(1)^2 \Rightarrow \boxed{y = \frac{1}{3}}$ عرض از مبدا