شادی حقیقی

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(1)

$3x^2 - ax + 4 = 0$ $\quad \beta \Rightarrow \frac{2}{3} \to 3\times\frac{4}{9} - \frac{2}{3}a + 4 = 0 \xrightarrow{\times 3} 4 - 2a + 12 = 0 \quad \boxed{a = 8}$

$\beta, 3\beta \xrightarrow{\times} 3\beta^2 = \frac{4}{3} \to \beta = \pm\frac{2}{3}$ $\quad \beta \Rightarrow -\frac{2}{3} \to 3\times\frac{4}{9} + \frac{2}{3}a + 4 \xrightarrow{\times 3} \quad \boxed{a = -8}$

اختلاف مقادیر: ۱۶

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(2)

$36x^2 - (m+14)x + 1 = 0$ $\quad \alpha\beta = \frac{1}{36}$ $\quad \alpha + \beta = \frac{m+14}{36}$

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5 \to \frac{\sqrt{\beta}+\sqrt{\alpha}}{\sqrt{\alpha\beta}\ \frac{1}{6}} = 5 \quad \frac{5}{6}$ $\quad \to (\sqrt{\beta}+\sqrt{a})^2 = \alpha + \beta + 2\sqrt{\alpha\beta}$

$\frac{25}{36} = \frac{m+14}{36} + \frac{12}{36}$

$\frac{13}{36} \to \boxed{m = -1}$

$mx^2 + 3x + 2 \xrightarrow{m=-1}$ حاصلضرب $= -2$

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(3)

$4x^3 + kx^2 - 9x - 2 = 0$ $\quad \alpha + \beta = 1 \quad \alpha\beta = -2$

$x^2 - x - 2 = 0 \to x = 2, \quad x = -1$

$x = -1 \to (4\times -1) + k + 9 - 2 = 0$

$\underbrace{-4 + 9 - 2}_{5} + k = 0 \to \boxed{k = -3}$

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(4)

$x^2 + 6x + a = 0$ $\quad \alpha < \beta < 0$ $\quad 3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85$ $\quad a = ?$

از ۳ و ۲ میانگین گرفتم

$3\alpha^2 + 2\beta^2 = 2.5(\alpha^2+\beta^2) + 0.5(\alpha^2 - \beta^2) \Rightarrow$

$2.5(36 - 2a) + 0.5[(-6)(-2\sqrt{9-a})] = 6\sqrt{9-a} + 90 - 5a = 12\sqrt{2} + 85$

$-5a + 6\sqrt{9-a} = -5 + 12\sqrt{2} \to \boxed{a = 1}$

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(5)

$x^4 - 7x^2 - 5 = 0$ $\quad x^2 = z$ $\quad S = 0$

$z^2 - 7z - 5 = 0 \to \Delta = 69 \to z = \frac{7 \pm \sqrt{69}}{2} \to x^2 = \frac{7+\sqrt{69}}{2}$

$x_1 = -\sqrt{\frac{7+\sqrt{69}}{2}}$ $\quad x_2 = \sqrt{\frac{7+\sqrt{69}}{2}}$

$P = x_2 x_1 = \frac{7+\sqrt{69}}{2} \to P^2 = \frac{59 + 7\sqrt{69}}{2} \to \boxed{2P^2 = 59 + 7\sqrt{69}}$

$2P^2 - 3\times 0 \times P + 0 = 2P^2 \leftarrow S = 0$ چون

(6)

$2x^2-ax+b=0 \quad (\alpha+\frac12),(\beta+\frac12)$ $\xrightarrow{\text{ضرب}}$ $\alpha\beta+\frac12(\alpha+\beta)+\frac14=\frac b2 \to -3-\frac14+\frac14=\frac b2$ $\boxed{b=-6}$

$\to \alpha+\beta+1=\frac a2 \to \boxed{a=1}$

$2ax^2+ax-6=0 \quad \alpha,\beta \to \alpha\beta=-\frac3a=-3 \ ,\ \alpha+\beta=-\frac12$

$[\frac{ab}{4}]=-2$

(7)

$a(x^2-x)-b=0$

$ax^2-ax-b=0$

$\alpha\beta=-\frac ba \quad \alpha+\beta=1$

$(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$

$1=\alpha^2+\beta^2-\frac{2b}{a}$

$40\beta^2+20\alpha^2-20\beta=17 \xrightarrow{\alpha=1-\beta} 40\beta^2+20+20\beta^2-40\beta=17$ $\quad 60\beta^2-60\beta+3=0$

$a(x^2-x)-b=0 \xrightarrow{x=\beta} a(\beta^2-\beta)-b=0 \to$

$a(-\frac{3}{60})=b \to -\frac ba=\frac{1}{20}=\alpha\beta$

$|\beta-\alpha|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\frac{2}{\sqrt5}$

(8)

$x^2-(a+1)x+a=0 \xrightarrow{\text{جمع ضرایب ۰}} 1, \alpha \xrightarrow{S=4} 3$

$x^2-4x+3 \qquad P=3$

$x^2-(3a+1)x+b=0$

$x^2-10x+b=0 \to S=10 \to 4,\ 6 \to b=24 \qquad P=24$

$\Rightarrow 24$ الف

(9)

$x^2+x-1-m^2=0 \quad \alpha^2+\beta^2=?$

$\alpha\beta=-1-m^2$

$\alpha+\beta=-1 \to (\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$

$1=\alpha^2+\beta^2-2-2m^2 \to \alpha^2+\beta^2=3+2m^2 \to m=0$

$\min=3$

(10)

$x_S=\frac{x_A+x_B}{2}=-1 \qquad y=1 \qquad y=a(x+1)^2+1 \to ax^2+2ax+(a+1)=0$

$\alpha+\beta=-2 \qquad \alpha\beta=\frac{a+1}{a}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=5=4-2(\frac{a+1}{a}) \to a=-\frac23$

$y=-\frac23(x+1)^2+1 \xrightarrow{\text{به ازای ۰}} y=-\frac23+1=\frac13$