① $x_1=\alpha$, $x_2=3\alpha$

$P=\frac{c}{a}=\frac{4}{3}=3\alpha^2\to\alpha^2=\frac{4}{9}\to\alpha=\pm\frac{2}{3}$

$\alpha \to \alpha=\frac{2}{3}\Rightarrow x_1=\frac{2}{3}\quad x_2=2\Rightarrow S=\frac{8}{3}=\frac{8}{3}\to a=8$

$\to \alpha=-\frac{2}{3}\Rightarrow x_1=-\frac{2}{3}\quad x_2=-2\Rightarrow S=\frac{-8}{3}=-\frac{8}{3}\to a=-8$

$|8-(-8)|=16$

---

② $\alpha+\beta=\frac{m+14}{36}\quad \alpha\beta=\frac{1}{36}$

$\sqrt{\frac{1}{\alpha}}+\sqrt{\frac{1}{\beta}}=5\to\frac{1}{\alpha}+\frac{1}{\beta}+2\sqrt{\frac{1}{\alpha\beta}}=25\to\frac{\alpha+\beta}{\alpha\beta}+2\sqrt{36}=25$

$m+14+12=25\to m=-1\Rightarrow -x^2+3x+2=0\Rightarrow x=\frac{-3\pm\sqrt{9+8}}{-2}$

$\frac{-3+\sqrt{17}}{2}\times\frac{-3-\sqrt{17}}{-2}=\frac{9-17}{4}=-2$

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③ (معادله جدید برای بخش پذیری) $\alpha\beta=-2\quad \alpha+\beta=1\to x^2-x-2=0$

$$\begin{array}{l|l} 4x^3+kx^2-9x-2 & x^2-x-2 \\ \underline{4x^3-4x^2-8x} & 4x+1 \\ (k+4)x^2-x-2 & \\ \underline{(1)x^2-x-2} & \\ 0 & \end{array}$$

$\Rightarrow k+4=1\to\boxed{k=-3}$

---

④ $2.5\alpha^2+2.5\beta^2+(0.5\alpha^2-0.5\beta^2)=2.5(\alpha^2+\beta^2)+0.5(\alpha^2-\beta^2)$

$2.5(36-2a)+\frac{1}{2}((-6)(-2\sqrt{9-a}))=90-5a+6\sqrt{9-a}=85+12\sqrt{2}$

$-5a+6\sqrt{9-a}=-5+12\sqrt{2}\Rightarrow\boxed{a=1}$

---

⑤ $x^2=t\to t^2-7t-5=0\to t\Rightarrow\frac{7+\sqrt{69}}{2}\ \checkmark\ x^2$

$\frac{7-\sqrt{69}}{2}\Rightarrow$ غ ق

$x\pm\sqrt{\frac{7+\sqrt{69}}{2}}\to S=0\qquad P=\frac{-7+\sqrt{69}}{2}$

$2P^2-3\cancel{SP}+\cancel{2S}=2\left(\frac{49+69+14\sqrt{69}}{4}\right)=59+7\sqrt{69}$

(6)

$2ax^2+ax-6=0$

$x_1=\alpha \qquad x_2=\beta$

$S=\frac{a}{-2a}=\alpha+\beta\to\alpha+\beta=-\frac{1}{2}$

$2x^2+x-6=0\to\alpha=-2\quad\beta=\frac{3}{2}$

$2x^2-ax+b=0$

$x_1=\alpha+\frac{1}{2} \qquad x_2=\beta+\frac{1}{2}$

$S=\alpha+\beta+1=\frac{a}{2}$

$-\frac{1}{2}+\frac{2}{2}=\frac{a}{2}\Rightarrow a=1$

$\Rightarrow \alpha=-\frac{3}{2} \qquad \beta=2$

$a=1 \qquad b=-6$

$\left[\frac{ab}{4}\right]=\left[-\frac{6}{4}\right]=\boxed{-2}$

---

(7)

$\alpha+\beta=1 \quad \alpha\cdot\beta=-\frac{b}{a}\to x^2-x-\frac{b}{a}=0\to x^2-x=\frac{b}{a}\to\beta^2-\beta=\frac{b}{a}=t$

$20\alpha^2+20\beta^2+20\beta^2-20\beta=20\left(1+\frac{2b}{a}\right)+20\left(\frac{b}{a}\right)=20+60t=17$

$60t=-3\to\frac{b}{a}=-\frac{1}{20}$

$\alpha-\beta=\frac{\sqrt{\Delta}}{|a|}=\sqrt{1+4\frac{b}{a}}=\sqrt{\frac{4}{5}}=\boxed{\frac{2\sqrt{5}}{5}}$

---

(8)

$x^2-(a+1)x+a=0$

$x_1=2\alpha-1 \qquad x_2=2\alpha+1$

$P=4\alpha^2-1=a$

$S=4\alpha=a+1$ } $a\neq1$ ↳ ۰ غ ق

$\Rightarrow\boxed{a=3}$

$P=3$

$24-3=\boxed{21}$

$x^2-(3a+1)x+b=0$

$x_1'=2\beta \qquad x_2'=2\beta+2$

$S'=(3(3)+1)=4\beta+2\to\beta=2$

$P=4\beta^2+4\beta=b\to\boxed{b=24}$

$P'=24$

---

(9)

$(\alpha+\beta)^2-2\alpha\beta=(-1)^2+2(m^2+1)=1+2m^2+2=2m^2+3=\boxed{3}$ → کمترین مقدار

باید ۰ بشه

$\boxed{\alpha+\beta=-1}$ $\boxed{\alpha\beta=-m^2-1}$

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(10)

$\frac{-b}{2a}=\frac{S}{2}=-1\to S=-2 \qquad S^2-2P=4-2\left(\frac{a+1}{a}\right)$

$a=-\frac{2}{3} \qquad S^2-2P=4-2P=5\to P=\frac{c}{a}=-\frac{1}{2}$

$\frac{c}{a}=-\frac{1}{2}\to c=\frac{-a}{2}=\frac{\frac{2}{3}}{2}=\boxed{\frac{1}{3}}$