

$$x > 1 \Rightarrow \frac{1}{x-1} = x-1 \Rightarrow 1 = (x-1)^2 = x^2 - 2x + 1 \Rightarrow x^2 - 2x = 0 = x(x-2) \Rightarrow x=2, x=0$$

$$x \leq -1 \Rightarrow \frac{1}{x-1} = 1-x \Rightarrow 1 = (x-1)(1-x) = 2x - 1 - x^2 \Rightarrow x^2 - 2x + 2 = 0 \rightarrow \Delta = 4 - 4(2) = -4 < 0$$

لذا جواب ندارد.

پس: $x=2$

الف) $\frac{2x-1}{x} < 2 \Rightarrow \frac{2x-1}{x} - 2 < 0 \Rightarrow \frac{2x-1-2x}{x} = -\frac{1}{x} < 0 \Rightarrow x > 0$

$\frac{2x-1}{x} > -2 \Rightarrow \frac{2x-1}{x} + 2 > 0 \Rightarrow \frac{2x-1+2x}{x} > 0 \Rightarrow \frac{4x-1}{x} > 0$

$\Rightarrow (0, +\infty) \cap \left(\left(\frac{1}{4}, +\infty \right) \cup (-\infty, 0) \right) = \left(\frac{1}{4}, +\infty \right)$

ب) $\frac{x-2}{2x+1} > 1 \Rightarrow \frac{x-2}{2x+1} - 1 > 0 \Rightarrow \frac{x-2-2x-1}{2x+1} > 0 \Rightarrow \frac{-x-3}{2x+1} > 0$

$\frac{x-2}{2x+1} < -1 \Rightarrow \frac{x-2}{2x+1} + 1 < 0 \Rightarrow \frac{x-2+2x+1}{2x+1} < 0 \Rightarrow \frac{3x-1}{2x+1} < 0$

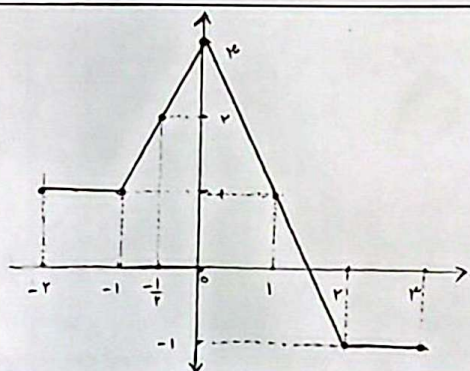
$\Rightarrow (-3, -\frac{1}{2}) \cup (-\frac{1}{2}, \frac{1}{3})$

$x+4 > x^2 \Rightarrow x^2 - x - 4 < 0 \Rightarrow (x-3)(x+2) < 0 \Rightarrow (-2, 3)$

$x+4 < -x^2 \Rightarrow x^2 + x + 4 < 0 \Rightarrow \Delta < 0 \rightarrow$ جواب ندارد

$\Rightarrow -2 < x < 3$

$\Rightarrow |x - \frac{1}{2}| < \frac{5}{2} \Rightarrow x = \frac{1}{2}$



نقاط قبل و بعد از این ها عدد مطلق را پیدا کنیم.

x	-2	-1	-1/4	0	1	2	3
y	1	1	2	3	1	-1	-1

$3-1=2$

حداکثر مقدار: $y = -1$

بیشترین مقدار: $y = 3$

$y = |\frac{1}{4}x| - 2 = \frac{1}{4}|x| - 2 \xrightarrow{\text{دو طرف را در ۴ ضرب}} y = \frac{1}{4}|x+4| - 2 \rightarrow y = |\frac{1}{4}x+2| - 2$

$\xrightarrow{\text{دو طرف را در ۴ ضرب}} y = |\frac{1}{4}x+2| - 2+1 = |\frac{1}{4}x+2| - 1$

$|\frac{1}{4}x+2| - 1 = |\frac{1}{4}x| - 2 \Rightarrow |\frac{1}{4}x+2|+1 = |\frac{1}{4}x|$

$\downarrow$ $\downarrow$
 همیشه -4 $\downarrow$ $\downarrow$ $\downarrow$
 $\rightarrow (-3, \frac{3}{4}) \rightarrow$ $x = -3$

$\begin{cases} x > 0 \rightarrow \frac{1}{4}x+2+1 = \frac{1}{4}x \Rightarrow 3=0 \text{ غلط} \\ -4 \leq x \leq 0 \rightarrow \frac{1}{4}x+2+1 = -\frac{1}{4}x \Rightarrow x = -3 \text{ قق} \\ x < -4 \rightarrow -\frac{1}{4}x-2+1 = -\frac{1}{4}x \Rightarrow -1=0 \text{ غلط} \end{cases}$

الف) $\begin{cases} x > 1 & -x-1-x-6 = -\infty \rightarrow f(x) = -\infty \\ -1 \leq x \leq 1 & 1-x-x-6 = -2x-5 \rightarrow -2 \leq -2x \leq 8 \rightarrow -4 \leq -2x-5 \leq 9 \\ x < -1 & 1-x+x+6 = 7 \rightarrow f(x) = 7 \end{cases} \Rightarrow f(x) = \{-\infty, -4, \dots, 7, 9\}$

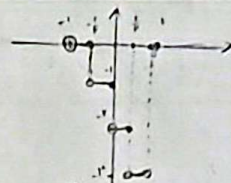
$\Rightarrow R_f = \{-\infty, -4, -2, \dots, 6, 9\}$

ب) $f(x) = \frac{1}{x} - \left[1 + \frac{1}{x}\right] = \frac{1}{x} - \left[\frac{1}{x}\right] - 1 \rightarrow \frac{1}{x} = t \Rightarrow 0 < \frac{1}{x} - \left[\frac{1}{x}\right] < 1$

$\Rightarrow -1 \leq \frac{1}{x} - \left[\frac{1}{x}\right] + 1 < 2 \Rightarrow R_f = [-1, 2)$

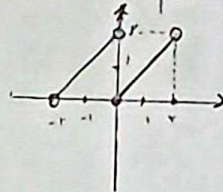
الف) $-1 < x < 1 \Rightarrow -2 < -2x < 2$

$$\begin{cases} -2 < -2x < -1 \rightarrow y = -2-1 = -3 \rightarrow \frac{1}{2} < x < 1 \\ -1 \leq -2x < 0 \rightarrow y = -1-1 = -2 \rightarrow 0 < x \leq \frac{1}{2} \\ 0 \leq -2x < 1 \rightarrow y = 0-1 = -1 \rightarrow -\frac{1}{2} < x \leq 0 \\ 1 \leq -2x < 2 \rightarrow y = 1-1 = 0 \rightarrow -1 < x \leq -\frac{1}{2} \end{cases}$$

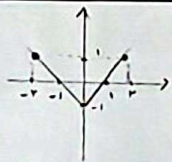


ب) $-2 < x < 2 \Rightarrow -1 < \frac{x}{2} < 1$

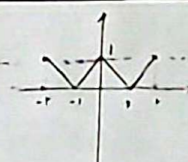
$$\begin{cases} -1 < \frac{x}{2} < 0 \rightarrow y = x-2(-1) = x+2 \rightarrow -2 < x < 0 \\ 0 \leq \frac{x}{2} < 1 \rightarrow y = x-2(0) = x \rightarrow 0 \leq x < 2 \end{cases}$$



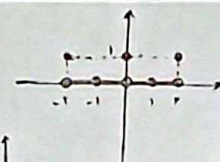
الف) $|x-1|$ نمودار



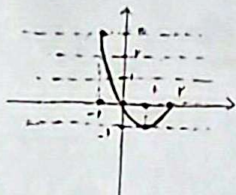
$|x-1|$ نمودار



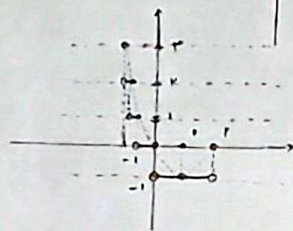
$[|x-1|]$ نمودار



ب) $x^2 - 2x$ نمودار



$[x^2 - 2x]$ نمودار



$a + [b] = 4, 2 \rightarrow a$ جزء اعشاری : $0, 2$

$b - [a] = 2, 4 \rightarrow b$ جزء اعشاری : $0, 4$

$\rightarrow [a] + [b] = 4$

$[b] - [a] = 2$

$\Rightarrow r[b] = 4 \Rightarrow [b] = 3 \Rightarrow b = 3 + 0, 4 = 3, 4$

$[a] = 4 - [b] = 1 \Rightarrow a = 1, 2$

$\rightarrow a + b = 1, 2 + 3, 4 = 4, 6$

$1 < \sqrt{2} < 2 \Rightarrow -2 < -\sqrt{2} < -1 \Rightarrow -1 < 1 - \sqrt{2} < 0 \Rightarrow 0 < (1 - \sqrt{2})^4 < 1$

$\rightarrow (1 + \sqrt{2})^4 + (1 - \sqrt{2})^4 = 198 \xrightarrow{0 < (1 - \sqrt{2})^4 < 1} \Rightarrow 197 < (1 + \sqrt{2})^4 < 199$

$\Rightarrow [(1 + \sqrt{2})^4] = 197$