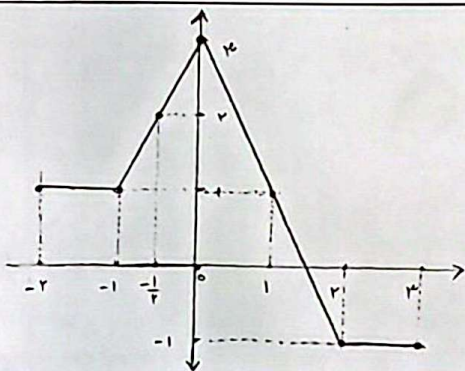


$$\begin{aligned}
 x > 1 &\Rightarrow \frac{1}{x-1} = x-1 \Rightarrow 1 = (x-1)^2 = x^2 - 2x + 1 \Rightarrow x^2 - 2x = 0 = x(x-2) \\
 &\Rightarrow \frac{x=2}{\text{دو}} , \frac{x=0}{\text{دو}} \\
 x \leq -1 &\Rightarrow \frac{1}{x-1} = 1-x \Rightarrow 1 = (x-1)(1-x) = 2x - 1 - x^2 \\
 &\Rightarrow x^2 - 2x + 2 = 0 \rightarrow \Delta = 4 - 4(2) = -4 < 0 \\
 &\quad \text{لذا جواب ندارد.} \\
 \text{پس نتیجه: } x &= 2
 \end{aligned}$$

$$\begin{aligned}
 \text{الف) } \frac{2x-1}{x} < 2 &\Rightarrow \frac{2x-1}{x} - 2 < 0 \Rightarrow \frac{2x-1-2x}{x} = -\frac{1}{x} < 0 \Rightarrow x > 0 \\
 \frac{2x-1}{x} > -2 &\Rightarrow \frac{2x-1}{x} + 2 > 0 \Rightarrow \frac{2x-1+2x}{x} > 0 \Rightarrow \frac{4x-1}{x} > 0 \\
 &\Rightarrow (0, +\infty) \cap \left(\left(\frac{1}{4}, +\infty \right) \cup (-\infty, 0) \right) = \left(\frac{1}{4}, +\infty \right) \\
 \text{ب) } \frac{x-2}{2x+1} > 1 &\Rightarrow \frac{x-2}{2x+1} - 1 > 0 \Rightarrow \frac{x-2-2x-1}{2x+1} > 0 \Rightarrow \frac{-x-3}{2x+1} > 0 \\
 &\quad \frac{-x-3}{2x+1} > 0 \Rightarrow \frac{-x-3}{2x+1} > 0 \Rightarrow \frac{-x-3}{2x+1} > 0 \Rightarrow \frac{-x-3}{2x+1} > 0 \Rightarrow \frac{-x-3}{2x+1} > 0 \\
 \frac{x-2}{2x+1} < -1 &\Rightarrow \frac{x-2}{2x+1} + 1 < 0 \Rightarrow \frac{x-2+2x+1}{2x+1} < 0 \Rightarrow \frac{3x-1}{2x+1} < 0 \\
 &\quad \frac{3x-1}{2x+1} < 0 \Rightarrow \frac{3x-1}{2x+1} < 0 \Rightarrow \frac{3x-1}{2x+1} < 0 \Rightarrow \frac{3x-1}{2x+1} < 0 \Rightarrow \frac{3x-1}{2x+1} < 0
 \end{aligned}$$

$$\begin{aligned}
 x+4 > x^2 &\Rightarrow x^2 - x - 4 < 0 \Rightarrow (x-3)(x+2) < 0 \Rightarrow \frac{-2}{-} \frac{3}{+} \Rightarrow (-2, 3) \\
 x+4 < -x^2 &\Rightarrow x^2 + x + 4 < 0 \Rightarrow \Delta < 0 \rightarrow \text{جواب ندارد} \\
 \Rightarrow -2 < x < 3 &\xrightarrow{\frac{x-2}{x} = \frac{1}{x}} -2 - \frac{1}{x} < x - \frac{1}{x} < 3 - \frac{1}{x} \Rightarrow -\frac{5}{x} < x - \frac{1}{x} < \frac{5}{x} \\
 \Rightarrow |x - \frac{1}{x}| < \frac{5}{x} &\Rightarrow x = \frac{1}{x}
 \end{aligned}$$



نقاط تقاطع و محورهای تقاطع را پیدا کنید.

x	-2	-1	-1/4	0	1	2	3
y	1	1	2	3	1	-1	-1

$$3 - 1 = 2$$

حداکثر مقدار: $y = -1$
حداقل مقدار: $y = 3$

$$y = \left| \frac{1}{4}x \right| - 2 = \frac{1}{4}|x| - 2 \xrightarrow{\text{دو طرفه ضرب}} y = \frac{1}{4}|x+4| - 2 \rightarrow y = \left| \frac{1}{4}x + 2 \right| - 2$$

$$\xrightarrow{\text{دو طرفه ضرب}} y = \left| \frac{1}{4}x + 2 \right| - 2 + 1 = \left| \frac{1}{4}x + 2 \right| - 1$$

$$\begin{aligned}
 \left| \frac{1}{4}x + 2 \right| - 1 &= \left| \frac{1}{4}x \right| - 2 \Rightarrow \left| \frac{1}{4}x + 2 \right| + 1 = \left| \frac{1}{4}x \right| \\
 &\quad \downarrow \quad \downarrow \\
 &\quad \text{پس نتیجه} \quad \text{پس نتیجه} \\
 &\rightarrow (-3, \frac{3}{4}) \rightarrow \text{تقاطع} \quad x = -3
 \end{aligned}$$

الف) $\begin{cases} x > 1 & -x-1-x-6 = -\infty \rightarrow f(x) = -\infty \\ -1 \leq x \leq 1 & 1-x-x-6 = -2x-5 \rightarrow -2 \leq -2x \leq 2 \rightarrow -4 \leq -2x-5 \leq 2 \\ x < -1 & 1-x+x+6 = 7 \rightarrow f(x) = 7 \end{cases} \Rightarrow f(x) = \{-\infty, -4, \dots, 6, \infty\}$

$\Rightarrow R_f = \{-\infty, -4, -2, \dots, 6, \infty\}$

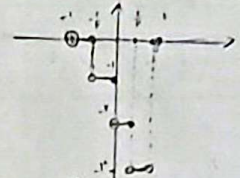
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ب) $f(x) = \frac{1}{x} - \left[1 + \frac{1}{x}\right] = \frac{1}{x} - \left[\frac{1}{x}\right] - 1 \rightarrow \frac{1}{x} = t \Rightarrow 0 < \frac{1}{x} - \left[\frac{1}{x}\right] < 1$

$\Rightarrow -1 \leq \frac{1}{x} - \left[\frac{1}{x}\right] + 1 < 2 \Rightarrow R_f = [-1, 2)$

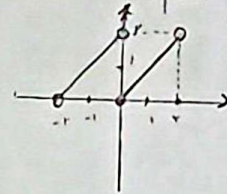
الف) $-1 < x < 1 \Rightarrow -2 < -2x < 2$

$$\begin{cases} -2 < -2x < -1 \rightarrow y = -2-1 = -3 \rightarrow \frac{1}{2} < x < 1 \\ -1 \leq -2x < 0 \rightarrow y = -1-1 = -2 \rightarrow 0 < x \leq \frac{1}{2} \\ 0 \leq -2x < 1 \rightarrow y = 0-1 = -1 \rightarrow -\frac{1}{2} < x \leq 0 \\ 1 \leq -2x < 2 \rightarrow y = 1-1 = 0 \rightarrow -1 < x \leq -\frac{1}{2} \end{cases}$$

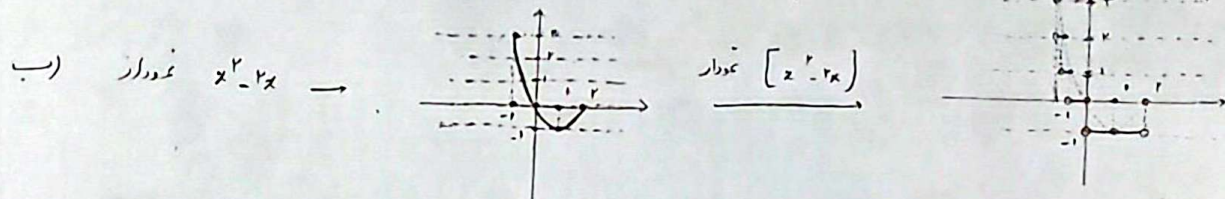
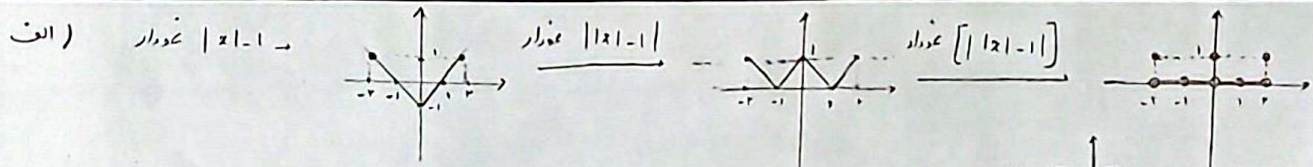


ب) $-2 < x < 2 \Rightarrow -1 < \frac{x}{2} < 1$

$$\begin{cases} -1 < \frac{x}{2} < 0 \rightarrow y = x - 2(-1) = x + 2 \rightarrow -2 < x < 0 \\ 0 \leq \frac{x}{2} < 1 \rightarrow y = x - 2(0) = x \rightarrow 0 \leq x < 2 \end{cases}$$



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$a + [b] = 4, 2 \rightarrow$ جزء اعشاری a : $0, 2$

$b - [a] = 2, 4 \rightarrow$ جزء اعشاری b : $0, 4$

$\rightarrow [a] + [b] = 4$

$[b] - [a] = 2 \Rightarrow r[b] = 4 \Rightarrow [b] = 3 \Rightarrow b = 3 + 0, 4 = 3, 4$

$[a] = 4 - [b] = 1 \Rightarrow a = 1, 2$

$\rightarrow a + b = 1, 2 + 3, 4 = 4, 4$

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$1 < \sqrt{2} < 2 \Rightarrow -2 < -\sqrt{2} < -1 \Rightarrow -1 < 1 - \sqrt{2} < 0 \Rightarrow 0 < (1 - \sqrt{2})^4 < 1$

$\rightarrow (1 + \sqrt{2})^4 + (1 - \sqrt{2})^4 = 198 \xrightarrow{0 < (1 - \sqrt{2})^4 < 1} 196 < (1 + \sqrt{2})^4 < 199$

$\Rightarrow [(1 + \sqrt{2})^4] = 196$

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