

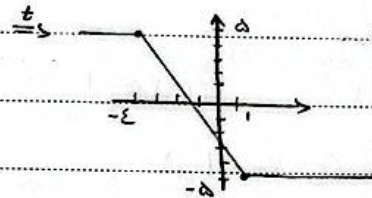
$$y' = \left| \frac{1}{y}x + 1 \right| - 1 \quad \left| \frac{1}{y}x + 1 \right| - 1 = \left| \frac{1}{y}x \right| - 1$$

$$y = \left| \frac{1}{y}x \right| - 1$$

$$\Rightarrow |x + 1| - 1 = |x| - 1 \Rightarrow \left| \frac{x+1}{1} \right| - \left| \frac{x}{1} \right| = -1$$

$$x - 1 - 1 - 1 = -1 \Rightarrow x = -1$$

$$f(x) = \left[\left| 1-x \right| - \left| x+1 \right| \right]$$



$$\Rightarrow f(x) = [t] \quad \text{حيث } t = \left| 1-x \right| - \left| x+1 \right|$$

$$R_f = \{1, 0, -1, -2, -3, \dots\}$$

$$f(x) = \frac{1}{x} - \left[\frac{x+1}{x} \right] = \frac{1}{x} - \left[1 + \frac{1}{x} \right] = \frac{1}{x} - 1 - \left[\frac{1}{x} \right]$$

$$0 < \frac{1}{x} - \left[\frac{1}{x} \right] < 1 \Rightarrow -1 < \frac{1}{x} - 1 - \left[\frac{1}{x} \right] < 0 \Rightarrow R_f = [-1, 0)$$

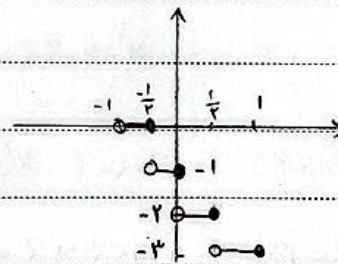
$$f(x) = [-2x] - 1 \quad x \in (-1, 1)$$

$$-1 < x < -\frac{1}{2} \rightarrow y = 0$$

$$-\frac{1}{2} < x < 0 \rightarrow y = -1$$

$$0 < x < \frac{1}{2} \rightarrow y = -2$$

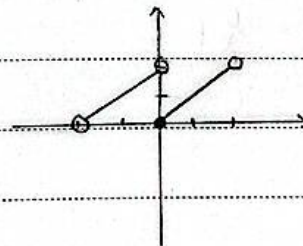
$$\frac{1}{2} < x < 1 \rightarrow y = -3$$



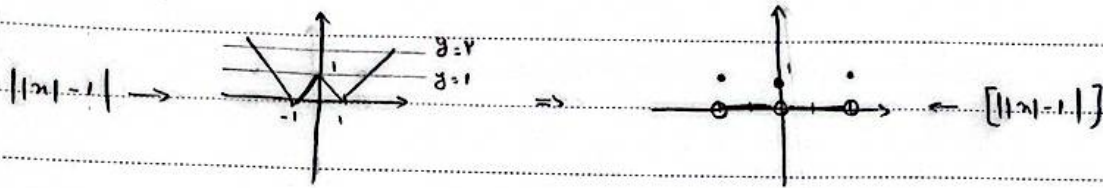
$$f(x) = 2 \left[\frac{x}{2} \right] \quad x \in (-2, 2)$$

$$-2 < x < 0 \rightarrow y = x + 2$$

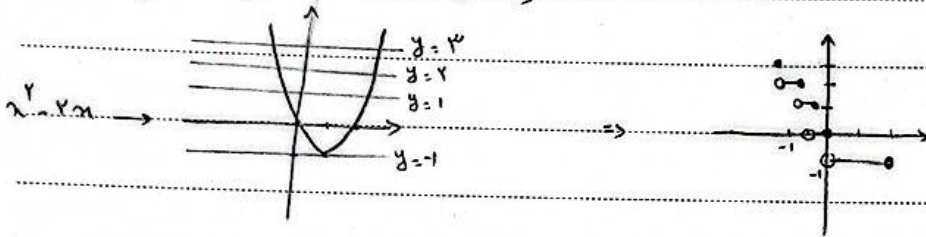
$$0 < x < 2 \rightarrow y = x$$



(vi) $y = [|x| - 1]$ $x \in [-2, 2]$



(vii) $y = [x^2 - 2x]$ $x \in [-1, 2]$



$a \in [b] = [2, 4] \rightarrow [a] = x \Rightarrow a = x + 0.5 \rightarrow x + 0.5 + y = 2, 4$

$b \in [a] = [2, 4] \rightarrow [b] = y \Rightarrow b = y + 0.5 \rightarrow y + 0.5 + x = 2, 4$

$x + y = 2$ $xy = 4$

$\Rightarrow y - x = 2 \rightarrow y = 3, x = 1 \Rightarrow a + b = x + 0.5 + y + 0.5 = 1 + 0.5 + 3 + 0.5 = 5, 4$

$-1 < 1 - \sqrt{2} < 0 \rightarrow 0 < (1 - \sqrt{2})^4 < 1 \rightarrow -1 < (1 - \sqrt{2})^4 < 0$ ①

$(1 + \sqrt{2})^4 = 19\sqrt{2} - (1 - \sqrt{2})^4$

② $19\sqrt{2} - 1 < (1 + \sqrt{2})^4 < 19\sqrt{2} \rightarrow 19\sqrt{2} < (1 + \sqrt{2})^4 < 19\sqrt{2} \Rightarrow [1 + \sqrt{2}] = 19\sqrt{2}$