

$$\frac{1}{x-1} = |x-1| \quad \textcircled{1} x > 1 \rightarrow \frac{1}{x-1} = x-1 \Rightarrow (x-1)^2 = 1 \Rightarrow x^2 - 2x + 1 = 1$$

$$\Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0 \rightarrow \begin{cases} x = 0 \times (x > 1) \\ x = 2 \checkmark \end{cases}$$

$$\textcircled{2} x < 1 \rightarrow \frac{1}{x-1} = 1-x \Rightarrow (x-1)(1-x) = 1 \Rightarrow -x^2 + 2x - 1 = 1$$

$$\Rightarrow -x^2 + 2x - 2 = 0 \Rightarrow \Delta < 0 \quad (\text{ریشه ندارد}) \quad \xrightarrow{f_1} x = 2$$

الف) $\left| \frac{x-1}{x} \right| < 2 \Rightarrow -2 < \frac{x-1}{x} < 2 \quad \textcircled{1} \frac{x-1}{x} < 2 \Rightarrow \frac{x-1-2x}{x} < 0 \Rightarrow \frac{-x-1}{x} < 0 \Rightarrow -x-1 < 0 \Rightarrow x > -1$

$$\textcircled{2} \frac{x-1}{x} > -2 \Rightarrow \frac{x-1+2x}{x} > 0 \Rightarrow \frac{3x-1}{x} > 0 \xrightarrow{f_2} \frac{3}{x} > 1 \Rightarrow x < 3$$

$$\xrightarrow{f_1} x \in (-1, 3)$$

ب) $\left| \frac{x-2}{x+1} \right| > 1 \Rightarrow \textcircled{1} \frac{x-2}{x+1} > 1 \Rightarrow \frac{x-2-x-1}{x+1} > 0 \Rightarrow \frac{-2x-3}{x+1} > 0 \Rightarrow \frac{-2}{x+1} > 0 \Rightarrow x < -1$

$$\textcircled{2} \frac{x-2}{x+1} < -1 \Rightarrow \frac{x-2+x+1}{x+1} < 0 \Rightarrow \frac{2x-1}{x+1} < 0 \xrightarrow{f_2} \frac{2}{x+1} < 1 \Rightarrow x > -1$$

$$\xrightarrow{f_1} x \in (-1, 3) \cup (-\infty, -1)$$

$$x^2 < |x+4| \Rightarrow \textcircled{1} x^2 < x+4 \Rightarrow x^2 - x - 4 < 0 \xrightarrow{f_1} \frac{-1 \pm \sqrt{17}}{2} \Rightarrow (-2, 3)$$

$$\textcircled{2} x^2 < x+4 \Rightarrow x^2 + x + 4 > 0 \Rightarrow \Delta < 0 \quad (\text{ریشه ندارد})$$

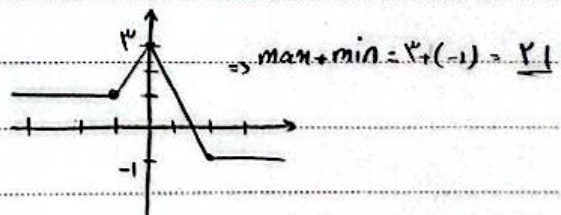
$$|x-\alpha| < \beta \Rightarrow -\beta < x-\alpha < \beta \Rightarrow \alpha-\beta < x < \alpha+\beta \sim -2 < x < 2$$

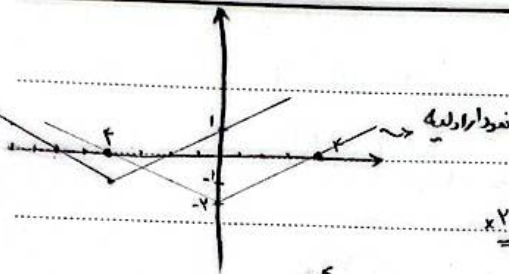
$$\Rightarrow \alpha - \beta = -2 \rightarrow 2\alpha = 1$$

$$\alpha + \beta = 2 \quad \alpha = \frac{1}{2}$$

$$y = \left| \frac{-1}{x+1} \right| - \left| \frac{0}{x} \right| + \left| \frac{2}{x-2} \right|$$

$-x-1+2x-x+2$	$x+1+2x-x+2$	$x+1-2x-x+2$	$x+1-2x+x-2$
$=1$	$=+2x+2$	$=-2x+2$	$=-1$





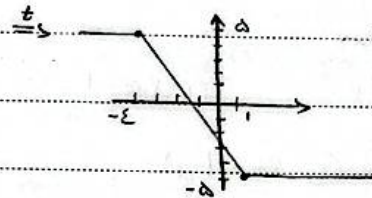
$$y' = \left| \frac{1}{2}x + 2 \right| - 1 \quad \left| \frac{1}{2}x + 2 \right| - 1 = \left| \frac{1}{2}x \right| - 2$$

$$y = \left| \frac{1}{2}x \right| - 2$$

$$\Rightarrow |x + 4| - 2 = |x| - 2 \Rightarrow \left| \frac{x+4}{2} \right| - \left| \frac{x}{2} \right| = -2$$

$$x - 2 \leq x \leq 2x + 4 \Rightarrow x = -4 \Rightarrow x = -4$$

$$f(x) = \left[\left| \frac{1}{2}x \right| - 2 \right]$$



$$\Rightarrow f(x) = [t] \quad \text{where } t = \frac{1}{2}x$$

$$R_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$$

$$f(x) = \frac{1}{x} - \left[\frac{x+1}{x} \right] = \frac{1}{x} - \left[1 + \frac{1}{x} \right] = \frac{1}{x} - 1 - \left[\frac{1}{x} \right]$$

$$0 \leq \frac{1}{x} - \left[\frac{1}{x} \right] < 1 \Rightarrow -1 \leq \frac{1}{x} - 1 - \left[\frac{1}{x} \right] < 0 \Rightarrow R_f = [-1, 0)$$

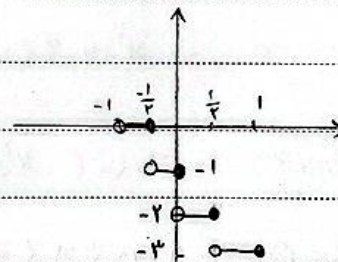
$$f(x) = [-2x] - 1 \quad x \in (-1, 1)$$

$$-1 < x \leq -\frac{1}{2} \rightarrow y = 0$$

$$-\frac{1}{2} < x \leq 0 \rightarrow y = -1$$

$$0 < x \leq \frac{1}{2} \rightarrow y = -2$$

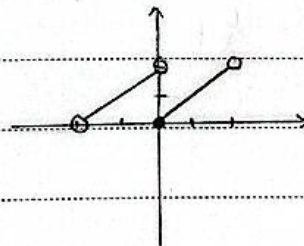
$$\frac{1}{2} < x \leq 1 \rightarrow y = -3$$



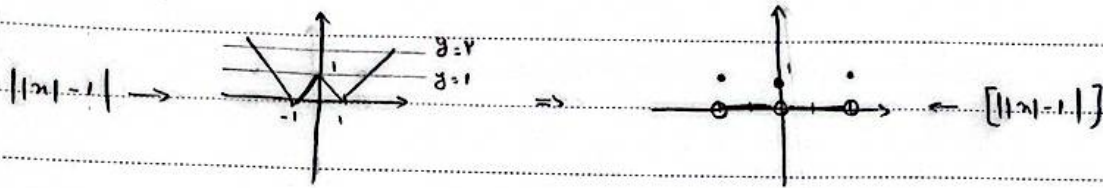
$$f(x) = 2 \left[\frac{x}{2} \right] \quad x \in (-2, 2)$$

$$-2 < x < 0 \rightarrow y = x + 2$$

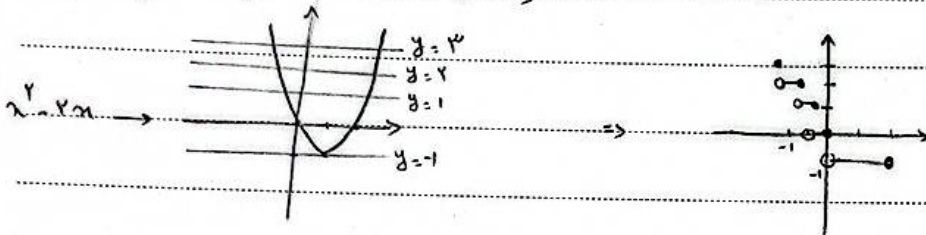
$$0 \leq x < 2 \rightarrow y = x$$



(vi) $y = [|x| - 1]$ $x \in [-2, 2]$



(vii) $y = [x^2 - 2x]$ $x \in [-1, 2]$



$a \in [b] = [2, 4] \rightarrow [a] = x \Rightarrow a = x + 0.5 \rightarrow x + 0.5 + y = [2, 4]$

$b \in [a] = [2, 4] \rightarrow [b] = y \Rightarrow b = y + 0.5 \rightarrow y + 0.5 + x = [2, 4]$

$x + y = 2 \rightarrow xy = 4$
 $\Rightarrow y - x = 2 \rightarrow y = 3, x = 1 \Rightarrow a + b = x + 0.5 + y + 0.5 = 1 + 0.5 + 3 + 0.5 = [5, 4]$

$-1 < 1 - \sqrt{2} < 0 \rightarrow 0 < (1 - \sqrt{2})^4 < 1 \rightarrow -1 < (1 - \sqrt{2})^4 < 0$ (1)

$(1 + \sqrt{2})^4 = 19\sqrt{2} - (1 - \sqrt{2})^4$

$\Rightarrow 19\sqrt{2} - 1 < (1 + \sqrt{2})^4 < 19\sqrt{2} \rightarrow 19\sqrt{2} < (1 + \sqrt{2})^4 < 19\sqrt{2} \Rightarrow [1 + \sqrt{2}] = 19\sqrt{2}$