

$$\frac{1}{x-1} = |x-1| \Rightarrow \frac{x^2 - 2x + 1}{(x-1)^2} = \frac{1}{x-1} \Rightarrow x - \frac{1}{x} = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$$t = (x-1)^2 = \pm 1 \Rightarrow x = \begin{cases} \sqrt{1} \\ 0 \text{ و } 2 \end{cases}$$

۱

الف) $\left| \frac{x-1}{x} \right| < 2$

$$\Rightarrow \frac{x^2 - 2x + 1}{x^2} < 2$$

$$x^2 - 2x + 1 < 2x^2$$

$$\Rightarrow -x^2 + 2x - 1 < 0$$

$$\Rightarrow x > \frac{1}{x}$$

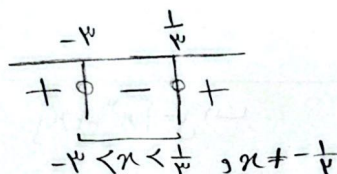
ب) $\left| \frac{x-1}{x+1} \right| > 1 \quad x \neq -\frac{1}{x}$

$$\frac{(x-1)^2}{(x+1)^2} > 1 \Rightarrow (x-1)^2 > (x+1)^2 \Rightarrow x^2 - 2x + 1 > x^2 + 2x + 1$$

$$\Rightarrow -4x > 0 \Rightarrow x < 0$$

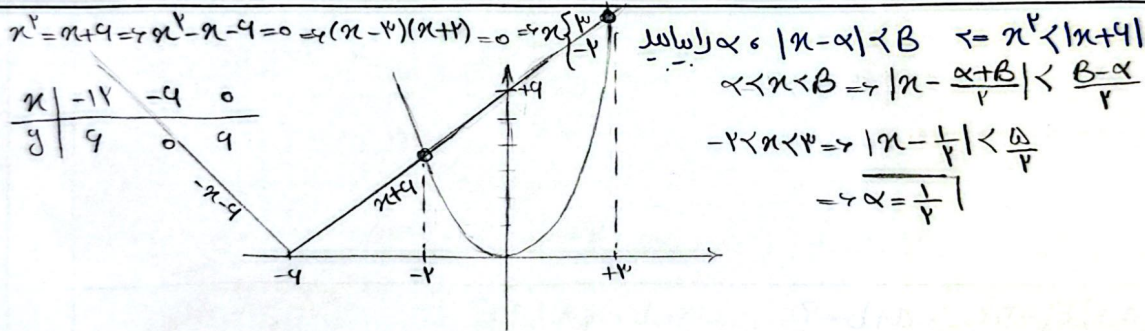
$$\Delta = 4 + 36 = 40$$

$$x = \frac{-4 \pm \sqrt{40}}{4} = \frac{-1 \pm \sqrt{10}}{1}$$



$$O_f = \left(-\frac{1}{\sqrt{10}}, -\frac{1}{\sqrt{10}} \right)$$

۲



۳

$x < -1 \Rightarrow y = x + 1 + 2x - x + 2 = +1$

مجموع مساحت دو مثلث: $y = |x+1| - |2x| + |x-2|$

x	-1	0	2
y	1	1	1

$-1 \leq x < 0 \Rightarrow y = x + 1 + |x - x + 2| = x + 1 + 2 = x + 3 \xrightarrow{x=0} y = 3$

$0 \leq x < 2 \Rightarrow y = x + 1 - 2x - x + 2 = -x + 3 \xrightarrow{x=0} y = 3$

$x > 2 \Rightarrow y = x + 1 - 2x + x - 2 = -1$

۴

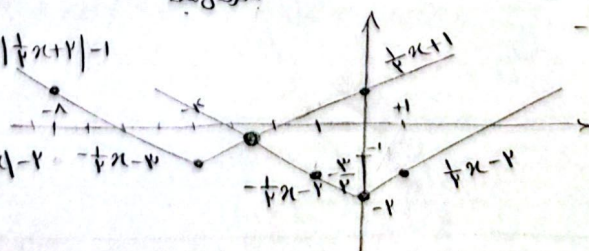
$y = \left| \frac{1}{2}x \right| - 2 \xrightarrow{\text{مقدار مطلق}} y = \left| \frac{1}{2}(x+2) \right| - 2 \xrightarrow{\text{مقدار مطلق}} y = \left| \frac{1}{2}x + 1 \right| - 2 + 1 = y = \left| \frac{1}{2}x + 1 \right| - 1$

x	0	-4	-8
y	1	-1	1

$$\Rightarrow y = \left| \frac{1}{2}x + 1 \right| - 1$$

x	-1	0	1
y	-1/2	-2	-3/2

$$\Rightarrow y = \left| \frac{1}{2}x \right| - 2$$



$$-\frac{1}{2}x - 2 = \frac{1}{2}x + 1$$

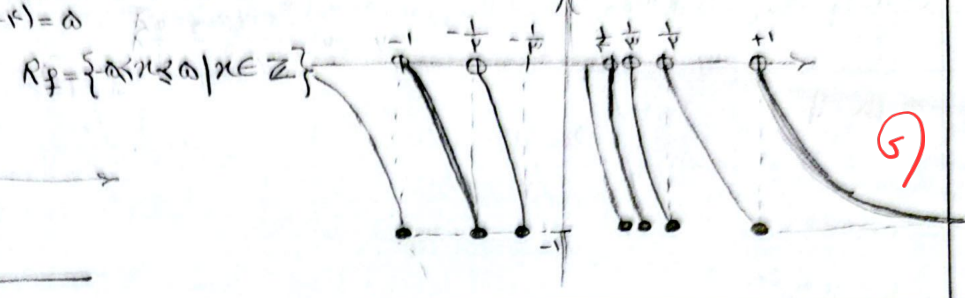
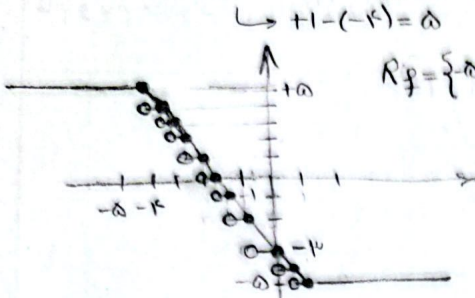
$$\Rightarrow x = -3$$

۵

$$الف) f(x) = \left[\frac{1-x}{1} - \frac{x+1}{-1} \right]$$

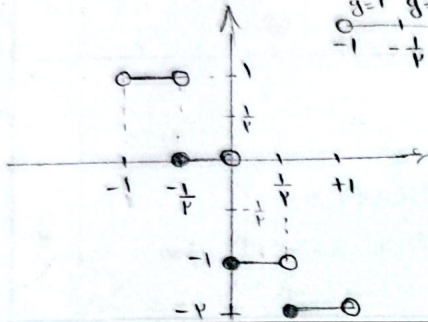
$$\frac{x}{y} \begin{array}{c|c|c|c|c|c} -1 & 0 & -1 & 0 & 1 & 1 \\ \hline 0 & 1 & 0 & -1 & 0 & -1 \end{array}$$

$$ب) f(x) = \frac{1}{x} - \left[\frac{x+1}{x} \right] = \frac{1}{x} - \left[\frac{1}{x} \right] - 1 \quad R_f = [-1, 0)$$



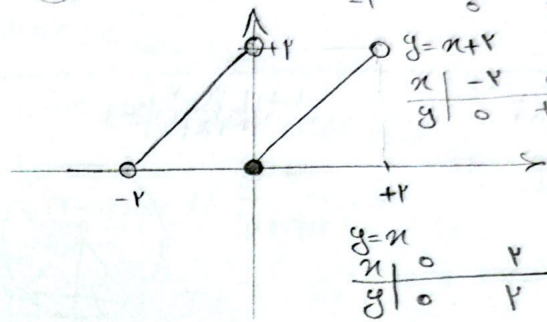
$$الف) g = [-2x] - 1 \quad x \in (-1, 1)$$

$$\frac{x}{y} \begin{array}{c|c|c|c|c|c} 1 & 0 & -1 & 0 & 1 & 1 \\ \hline -1 & -1 & 0 & 1 & 1 & -1 \end{array}$$

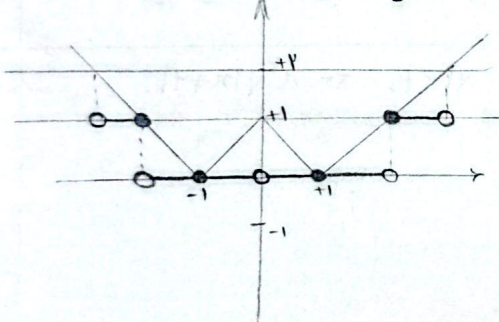


$$ب) x - \left[\frac{x}{y} \right] \quad x \in (-2, 2)$$

$$\frac{x}{y} \begin{array}{c|c|c|c|c|c} 2 & 1 & 0 & -1 & -2 & -2 \\ \hline -2 & -1 & 0 & 1 & 2 & 2 \end{array}$$

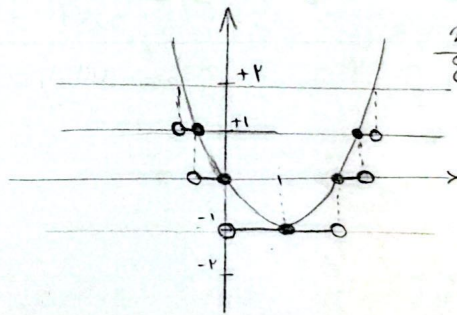


$$الف) [(1-x)-1] = y \quad x \in [-2, 2]$$



$$ب) g = [x^2 - 2x] \quad x \in [-1, 2] \quad \frac{-b}{2a} = \frac{1}{2} = 1$$

$$\frac{x}{y} \begin{array}{c|c|c|c|c|c} 0 & 1 & 2 & 0 & 0 \\ \hline 0 & -1 & 0 & 0 & 0 \end{array}$$



$$b - [a] = 2, 1, a + [b] = 1, 2, a + b = ? \quad a = b \Rightarrow [a] = [b]$$

$$\begin{aligned} [b - [a]] &= [2, 1] \\ [a + [b]] &= [1, 2] \end{aligned} \Rightarrow \begin{cases} [b] - [a] = 2 \\ [a] + [b] = 1 \end{cases} \Rightarrow \begin{cases} 2[b] = 4 \Rightarrow [b] = 2 \\ [a] = -1 \end{cases}$$

$$\begin{aligned} b - 1 &= 2, 1 \Rightarrow b = 3, 2 \\ a + 2 &= 1, 2 \Rightarrow a = -1, 0 \end{aligned} \Rightarrow a + b = 2, 1, 2, 2 \Rightarrow a = 1, 2$$

$$؟ (1+\sqrt{2})^4 \text{ كذا } (1-\sqrt{2})^4 + (1+\sqrt{2})^4 = 19$$

$$[(1+\sqrt{2})^4] = [19 - (1-\sqrt{2})^4]$$

$$\Rightarrow [(1+\sqrt{2})^4] = 19 + [-(1-\sqrt{2})^4] = 19 - 1 = 18$$

$$\begin{aligned} \frac{1}{1+\sqrt{2}} &= \frac{1-\sqrt{2}}{(1+\sqrt{2})(1-\sqrt{2})} \\ &= \frac{1-\sqrt{2}}{1-2} = \sqrt{2}-1 \end{aligned}$$