

$$\frac{1}{x-1} = |x-1| \begin{cases} x \geq 1 \\ x < 1 \end{cases} \Rightarrow \frac{1}{x-1} = x-1 \Rightarrow x^2 + (-2)x + 1 = 0 \Rightarrow x(x-2) = 0 \Rightarrow \begin{cases} x_1 = 2 \\ x_2 = 0 \end{cases}$$

جواب نهاده $\Delta = 4 - 4 = 0 \Rightarrow \Delta \leq 0$

$\Rightarrow x = 2 \Rightarrow$ ماده یک رسیده دارد

الف) $-2 < \frac{2x-1}{x} < 2$ $\Rightarrow \frac{2x-1}{x} < 2 \Rightarrow \frac{2x-1-2x}{x} < 0 \Rightarrow \frac{-1}{x} < 0 \Rightarrow x > 0$

$\Rightarrow \textcircled{1} \cap \textcircled{2} = \left(\frac{1}{2}, +\infty\right)$

$\textcircled{1} -2 < \frac{2x-1}{x} \Rightarrow \frac{2x-1-2x}{x} < 0 \Rightarrow \frac{-1}{x} < 0 \Rightarrow x > 0$

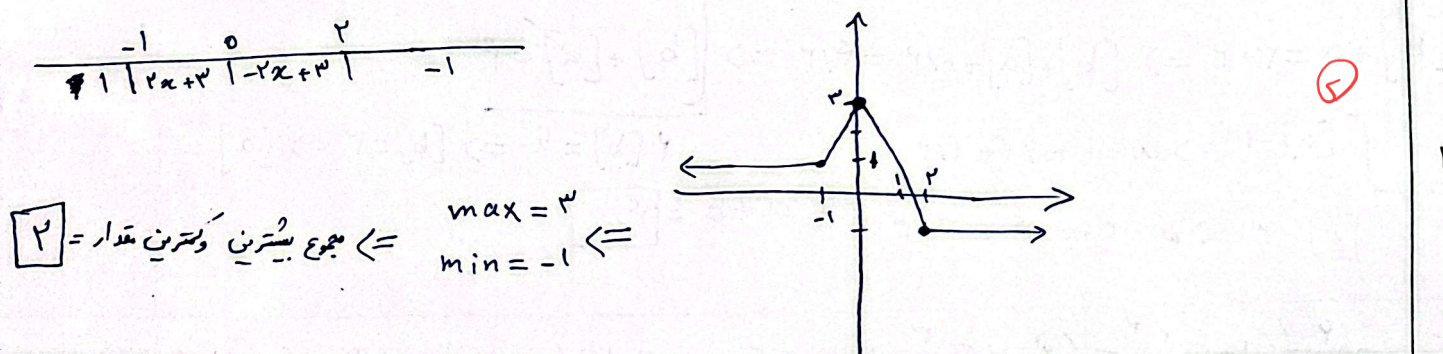
$\textcircled{2} \frac{2x-1}{x} < 2 \Rightarrow \frac{2x-1-2x}{x} < 0 \Rightarrow \frac{-1}{x} < 0 \Rightarrow x > 0$

$\Rightarrow \textcircled{1} \cup \textcircled{2} = \left(-\frac{1}{2}, \frac{1}{2}\right) - \left\{-\frac{1}{2}\right\}$

$$x^2 < |x+4| \begin{cases} x+4 > x^2 \\ x+4 < -x^2 \end{cases} \Rightarrow x^2 - x - 4 > 0 \Rightarrow -2 < x < 3 \textcircled{1}$$

جواب ندارد و به ازای $\Delta = 1 - 4 = -3 \Rightarrow$ هر دو عبارت مثبت است پس مجموعه جواب نامتناهی است.

$\textcircled{1} \cup \textcircled{2} \Rightarrow (-2 < x < 3)^{-\frac{1}{2}} \Rightarrow -\frac{1}{2} < x - \frac{1}{2} < \frac{1}{2} \Rightarrow |x - \frac{1}{2}| < \frac{1}{2} \Rightarrow x = \frac{1}{2}$



$$y' = \left| \frac{x+4}{x} \right| - 2 + 1 = \left| \frac{x+4}{x} \right| - 1$$

$y = y'$

$\left| \frac{x}{x} \right| - 2 = \left| \frac{x}{x} + 1 \right| - 1$

$\left| \frac{x}{x} + 1 \right| = \left| \frac{x}{x} \right| - 1 \Rightarrow \begin{cases} -\frac{x}{x} - 1 = -\frac{x}{x} - 1 \Rightarrow -2 = -1 \quad \times \quad ; x \leq -2 \\ \frac{x}{x} + 1 = -\frac{x}{x} - 1 \Rightarrow x = -3 \quad \text{غیر ممکن} \quad ; -2 < x < 0 \\ \frac{x}{x} + 1 = \frac{x}{x} - 1 \Rightarrow 2 = -1 \quad \times \quad ; x \geq 0 \end{cases}$

الف) $y = |1-x| - |x+4|$

$\frac{-4}{\omega} \mid \frac{1}{-2x-3} \mid \frac{-\omega}{-1} \Rightarrow f(x) = \begin{cases} \omega & x < -4 \\ [-2x]-3 & -4 \leq x < -1 \\ -\omega & x > -1 \end{cases}$

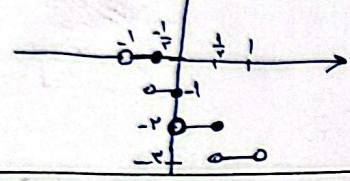
$\Rightarrow R_f = \{-\omega, -4, -3, -2, -1, 0, 1, 2, 3, 4, \omega\}$

$\begin{cases} \omega & x \in [-4, \frac{1}{2}) \\ 4 & x \in [-\frac{1}{2}, -3) \\ 3 & x \in [-3, -\frac{3}{2}) \\ 2 & x \in [-\frac{3}{2}, -2) \\ 1 & x \in [-2, -\frac{1}{2}) \\ 0 & x \in [-\frac{1}{2}, -\frac{1}{2}) \\ -1 & x \in [-\frac{1}{2}, -\frac{1}{2}) \\ -2 & x \in [-1, -\frac{1}{2}) \\ -3 & x \in [-\frac{1}{2}, 0) \\ -4 & x \in [0, \frac{1}{2}) \\ \omega & x \in [\frac{1}{2}, 1) \end{cases}$

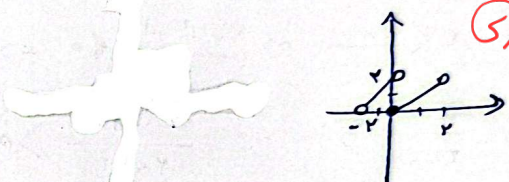
$\therefore f(x) = \frac{1}{x} - \left[\frac{x+1}{x} \right] = \frac{1}{x} - \left[1 + \frac{1}{x} \right] = \frac{1}{x} - \left[\frac{1}{x} \right] - 1$

$0 \leq \omega - [0] < 1 \Rightarrow (0 \leq \frac{1}{x} - \left[\frac{1}{x} \right] < 1) \Rightarrow -1 \leq \frac{1}{x} - \left[\frac{1}{x} \right] - 1 < 0 \Rightarrow R_f = [-1, 0)$

الف) $f(x) = \begin{cases} 0 & x \in (-1, -\frac{1}{2}] \\ -1 & x \in (-\frac{1}{2}, 0] \\ -2 & x \in (0, \frac{1}{2}] \\ -3 & x \in (\frac{1}{2}, 1) \end{cases}$



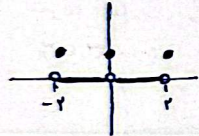
$\therefore f(x) = \begin{cases} x+1 & x \in (-2, 0) \\ x & x \in [0, 2) \end{cases}$



الف) $y = |x|-1 \Rightarrow$

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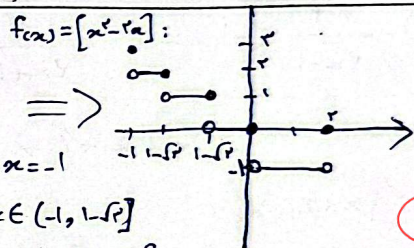


$\therefore y = x^2 - 2x \Rightarrow$

$\textcircled{1} x^2 - 2x = 2$
 $x = \pm \sqrt{2} + 1$

$\textcircled{2} x^2 - 2x = 1$
 $x = \pm \sqrt{2} + 1$

$f(x) = \begin{cases} \omega & x \in (-1, 1-\sqrt{2}] \\ 2 & x \in (1-\sqrt{2}, 1-\sqrt{2}] \\ 1 & x \in (1-\sqrt{2}, 1-\sqrt{2}] \\ 0 & x \in (1-\sqrt{2}, 0] \\ -1 & x \in (0, 2) \\ 0 & x \in 2 \end{cases}$



$b - [a] = 2/5 \Rightarrow [b] + 0/5 - [a] = 2/5 \Rightarrow [b] - [a] = 2/5$

$[b] + a = 5/2 \Rightarrow [b] + [a] + 0/2 = 5/2 \Rightarrow [b] + [a] = 5/2$

$\Rightarrow \begin{cases} [a] = 1 \Rightarrow a = 1 + 0/2 = 1/2 \\ [b] = 3 \Rightarrow b = 3 + 0/5 = 3/5 \end{cases} \Rightarrow a + b = \boxed{19/10}$

$(1+\sqrt{2})^4 = ((1+\sqrt{2})^2)^2 = (3+2\sqrt{2})^2 = 9 + 12\sqrt{2} + 8 = 17 + 12\sqrt{2}$

$\left[(1+\sqrt{2})^4 \right] = \left[17 + 12\sqrt{2} \right] = \left[12\sqrt{2} \right] + 17 = \left[\frac{12 \times 1.41}{1} \right] + 17 = 18 + 17 = \boxed{35}$