هلیا عماری 11B

①

$$\frac{1}{x-1}=|x-1|$$

$$\frac{1}{x-1}=-(x-1) \qquad \frac{1}{x-1}=x-1$$

$x>1$
$\rightarrow \frac{1}{x-1}=x-1 \rightarrow (x-1)^2=1 \rightarrow x^2-2x+1=1 \rightarrow x^2-2x=0 \rightarrow x=0$ غ ق ق ، $x=2$ ✓

$x<1$
$\rightarrow \frac{1}{x-1}=-(x-1) \rightarrow 1=-(x-1)^2 \rightarrow 1=-(x^2-2x+1) \rightarrow -x^2+2x-1=1$

$\rightarrow x^2-2x+2=0 \xrightarrow[\text{فاقد ریشه}]{\Delta<0}$

معادله فقط 1 ریشه $x=2$ دارد

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②

الف) $\left|\frac{2x-1}{x}\right|<2 \rightarrow -2<\frac{2x-1}{x}<2$ جواب $=(\frac{1}{4},+\infty)$

① $\frac{2x-1}{x}<2 \rightarrow \frac{2x-1}{x}-2<0 \rightarrow \frac{2x-1-\cancel{2x}}{x}<0 \rightarrow \frac{-1}{x}<0 \rightarrow \frac{1}{x}>0$

$\Rightarrow x>0$ *

② $-2<\frac{2x-1}{x} \rightarrow \frac{2x-1}{x}+2>0 \rightarrow \frac{2x-1+2x}{x}>0 \rightarrow \frac{4x-1}{x}>0$

$\rightarrow x=0,\ x=\frac{1}{4}$

	0		$\frac{1}{4}$	
+		−		+

$\rightarrow x<0 \cup x>\frac{1}{4}$ * $\xrightarrow[\text{اشتراک}]{*}$ $\boxed{x>\frac{1}{4}}$

ب) $\left|\frac{x-2}{2x+1}\right|>1$ $\nearrow \frac{x-2}{2x+1}>1$ ①   جواب ندارد
$\searrow \frac{x-2}{2x+1}<-1$ ②

$-(x+3)$

① $\frac{x-2}{2x+1}-1>0 \rightarrow \frac{x-2-2x-1}{2x+1}>0 \rightarrow \frac{-x-3}{2x+1}>0$

	-3		$-\frac{1}{2}$	
−		+		−

$x=(-3,-\frac{1}{2})$ *

② $\frac{x-2}{2x+1}+1<0 \rightarrow \frac{2x+1+x-2}{2x+1}<0 \rightarrow \frac{3x-1}{2x+1}<0$

	$-\frac{1}{2}$		$\frac{1}{3}$	
+		−		+

$x=(-\frac{1}{2},\frac{1}{3})$ *

$\xrightarrow{*} (-3,+\frac{1}{3})$

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③

$|x+6|\geq x^2$ $\nearrow x+6\geq x^2$ ① $\rightarrow x^2-x-6\leq 0 \rightarrow (x-3)(x+2)\leq 0$
$\searrow x+6\leq -x^2$ ② $\rightarrow x^2+x+6\leq 0 \xrightarrow[\text{همواره مثبت}]{\Delta<0}$ غ ق ق

	-2		3	
+		−		+

$\rightarrow -2\leq x\leq 3 \rightarrow |x-\frac{1}{2}|\leq\frac{5}{2} \rightarrow \boxed{\alpha=\frac{1}{2}}$

$\frac{3-2}{2}=0{,}5$

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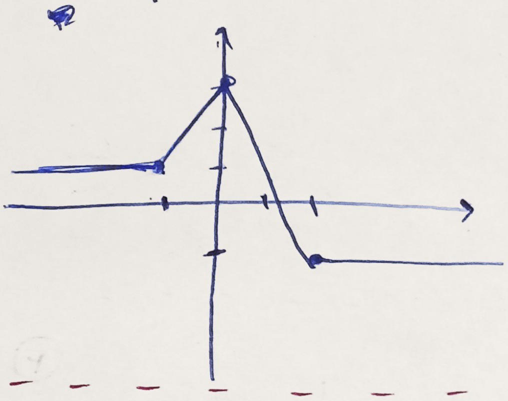
$-1 \qquad 0 \qquad 2$

$-x-1+2x$ | $x+1+2x-x+2$ | $x+1-2x-x+2$ | $x+1-2x+x-2$

$-x+2$ | $2x+3$ | $=-2x+3$ | $=-1$

$=1$



$$\left.\begin{array}{l} max=3 \\ min=-1 \end{array}\right\} max+min=2$$

$\frac{3}{2}-2-(\frac{3}{2}+2-1)$

$\frac{1}{2}x=-2$

$x=-4$

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$y=|\frac{1}{2}x|-2 \xrightarrow[\text{یکی واحدهای مثبت}]{\text{۴ تا واحدهای منفی}} y'=|\frac{1}{2}(x+4)|-2+1$

$|\frac{1}{2}x|-2=|\frac{1}{2}x+2|-1 \to |\frac{1}{2}x+2|-|\frac{1}{2}x|+1=0$

$-4 \qquad 0$

$-\frac{1}{2}x-2+\frac{1}{2}x+1$ | $\frac{1}{2}x+2+\frac{1}{2}x+1$ | $\frac{1}{2}x+2-\frac{1}{2}x+1$

$-1=0$ ✗ | $=x+3=0$ | $3\neq0$ ✗

$\boxed{\Rightarrow x=-3}$

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الف) $f(x)=[|1-x|-|x+4|]$

$-4 \qquad 1 \quad x>1$

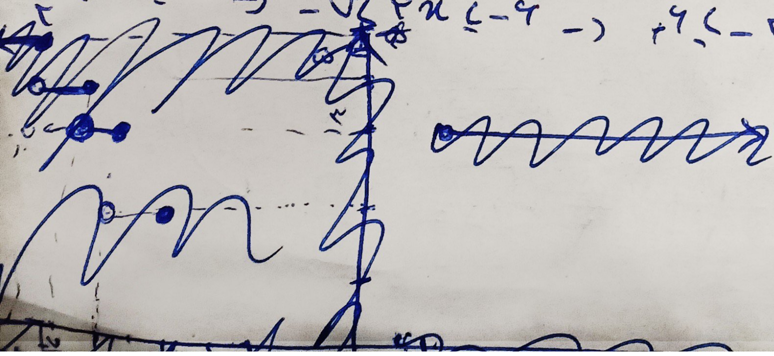
$1-x+x+4=\boxed{5}$ | $1-x-x-4=-2x-3 \to y=[-2x-3]=[-2x]-3$ | $x-1-x+4=\boxed{3}$

$-4<x\le-\frac{7}{2} \to -8<2x\le-7 \to 7\le-2x<8 \to [-2x]=7 \to y=7-3=\boxed{4}$

$-\frac{7}{2}<x\le-3 \to -7<2x\le-6 \to 6\le-2x<7 \to [-2x]=6 \to y=6-3=\boxed{3}$

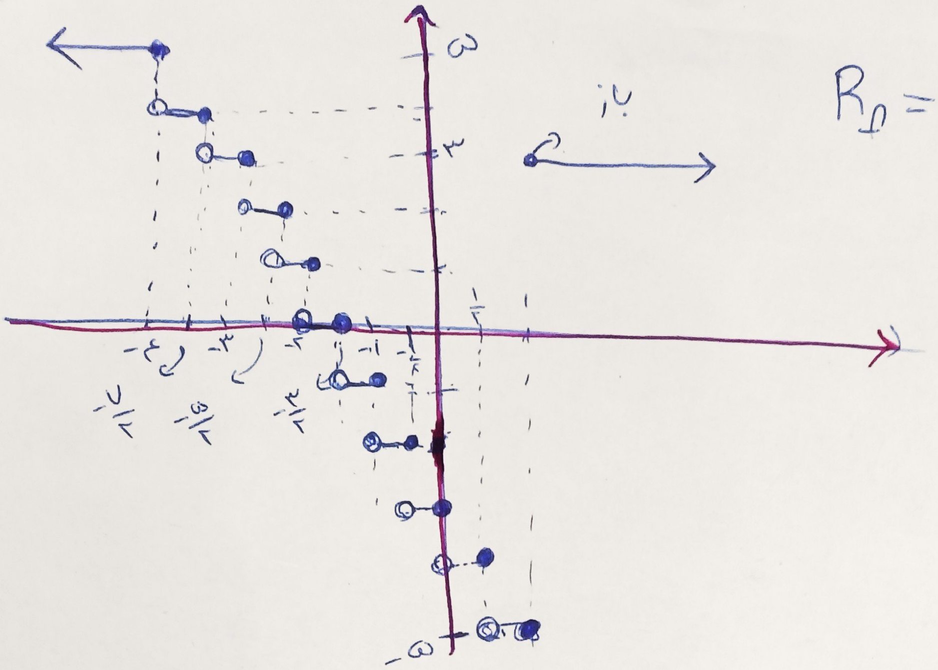
به ازای $\frac{1}{2}x$ های ۶ و ۷ یک واحد زیاد می‌شود.

نمودار صفحه بعد



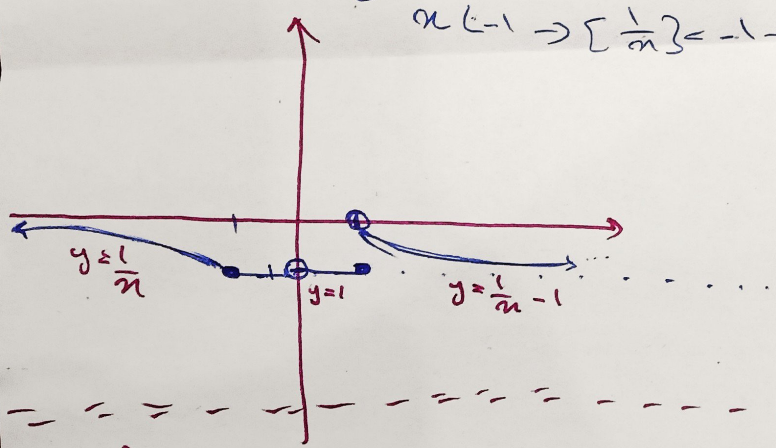
ادامه ۶

$R_f=\{x\in \mathbb{Z}, \mid -۵<x\le ۵\}$



ب) $f(x)=\frac{1}{x}-\left[\frac{x+1}{x}\right] \Rightarrow \frac{1}{x}-\left[1+\frac{1}{x}\right] \Rightarrow \frac{1}{x}-1-\left[\frac{1}{x}\right]$

$$\frac{1}{x}-\left[\frac{1}{x}\right]-1 \begin{cases} x>1 \to \left[\frac{1}{x}\right]=0, \quad y=\frac{1}{x}-1 \\ 0<x\le 1 \to \frac{1}{x}=\left[\frac{1}{x}\right] \Rightarrow y=-1 \\ -1\le x<0 \to \left[\frac{1}{x}\right]=\frac{1}{x} \to y=-1 \\ x<-1 \to \left[\frac{1}{x}\right]=-1 \to y=\frac{1}{x} \end{cases}$$



$R_f=[-1,0)$

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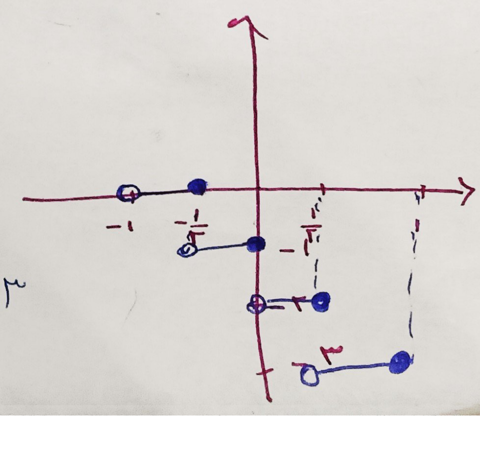
الف) $y=[-2x]-1 \quad x\in(-1,1)$

$-1<x\le-\frac{1}{2} \to -2<2x\le-1 \to 1\le-2x<2 \to [-2x]=1 \to y=1-1=0$

$-\frac{1}{2}<x\le 0 \to -1<2x\le 0 \to 0\le-2x<1 \to y=0-1=-1$; $[-2x]=0$

$0<x\le\frac{1}{2} \to 0<2x\le 1 \to -1\le-2x<0 \to y=-1-1=-2$; $[x]=-1$

$\frac{1}{2}<x\le 1 \to 1<2x\le 2 \to -2\le-2x<-1 \to y=-2-1=-3$; $[x]=-2$

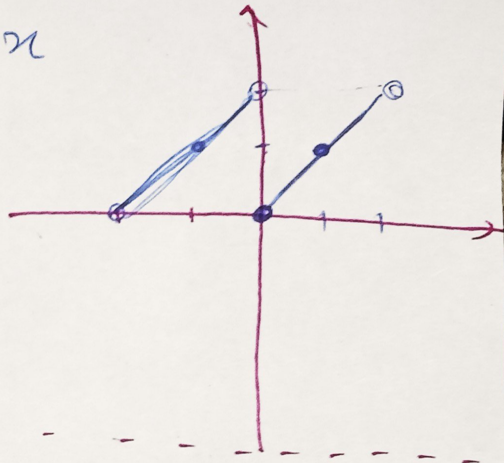


ادامه ۷

ب) $x-2\left[\frac{x}{2}\right]$ $\quad x\in(-2,2)$

$-2<x<0 \to -1<\frac{x}{2}<0 \to \left[\frac{x}{2}\right]=-1 \to y=x-2(-1)=x+2$

$0\leq x<2 \to 0\leq\frac{x}{2}<1 \to \left[\frac{x}{2}\right]=0 \to y=x$



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$$\left.\begin{array}{l} a+[b]=4{,}2 \\ b-[a]=2{,}4 \end{array}\right\} \to \begin{array}{l} a=n{,}2 \\ b=y{,}4 \end{array} \Rightarrow \begin{array}{l} a+b=4{,}6 \\ b-a=2{,}2 \end{array}$$

$2b=6{,}8$

$b=3{,}4$

$a+b+0{,}4=4{,}2$

$b-(a-0{,}2)=2{,}4$

$\boxed{a+b=4{,}6}$

$a+3{,}4=4{,}6$

$a=4{,}6-3{,}4=1{,}2$

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$(1+\sqrt{2})^4+\underbrace{(1-\sqrt{2})^4}_{(1-1{,}4)^4=(-0{,}4)^4}=198$

$=0<(0{,}4)^4<1$

$\Rightarrow (1+\sqrt{2})^4=198-(0{,}4)^4 \to 197<(1+\sqrt{2})^4<198$

$\Rightarrow \left[(1+\sqrt{2})^4\right]=\boxed{197}$