

$$\frac{1}{x-1} = |x-1|$$

$$x > 1 \rightarrow \frac{1}{x-1} = x-1 \rightarrow 1 = x^2 - 2x + 1 \rightarrow x(x-2) = 0 \rightarrow \begin{cases} x=2 \\ x=0 \end{cases}$$

(1)

$$x < 1 \rightarrow \frac{1}{x-1} = -(x-1) \rightarrow 1 = -x^2 + 2x - 1 \rightarrow \Delta < 0 \rightarrow \text{No solution}$$

(1) نتیجه دارد

$$\left| \frac{x-1}{x} \right| < 2$$

$$① \frac{x-1}{x} < 2 \rightarrow x-1 < 2x \rightarrow -1 < x \rightarrow x > -1$$

$$② \frac{x-1}{x} > -2 \rightarrow x-1 > -2x \rightarrow -1 > -3x \rightarrow x > \frac{1}{3}$$

$$\text{① و ②} \rightarrow \left(\frac{1}{3}, +\infty \right)$$

الف

$$\left| \frac{x-2}{x+1} \right| > 1$$

$$\begin{aligned} ① \frac{x-2}{x+1} > 1 &\rightarrow x-2 > x+1 \rightarrow -2 > 1 \rightarrow \text{No solution} \\ ② \frac{x-2}{x+1} < -1 &\rightarrow x-2 < -x-1 \rightarrow 2x < 1 \rightarrow x < \frac{1}{2} \end{aligned}$$

ب

$$x^2 < |x+4|$$

$$① x^2 < x+4 \rightarrow x^2 - x - 4 < 0 \rightarrow (x-3)(x+2) < 0 \rightarrow (-2, 3) \rightarrow \left. \begin{aligned} \frac{3+(-2)}{2} &= \frac{1}{2} \\ \frac{3+2}{2} &= \frac{5}{2} \end{aligned} \right\}$$

$$② x^2 > x+4 \rightarrow x^2 - x - 4 > 0 \rightarrow \Delta < 0 \rightarrow \text{No solution}$$

$$|x - \frac{1}{2}| < \frac{5}{2}$$

$$B = \frac{1}{2} \quad |x = \frac{1}{2}$$

$$y = |x+1| - |2x| + |x-2|$$

$$x > 2 \rightarrow x+1 - 2x + x-2 \rightarrow y = -1$$

$$-1 < x < 2 \rightarrow x+1 - 2x - x+2 \rightarrow y = -2x+3 \xrightarrow{[0,2]} (-1, 3)$$

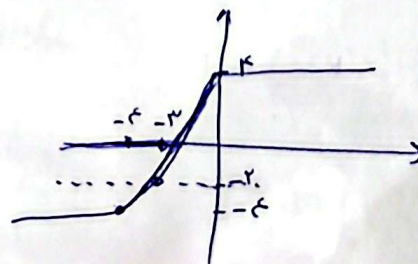
$$x < -1 \rightarrow x+1 + 2x - x+2 \rightarrow y = 2x+3 \xrightarrow{(-\infty, 0]} (-1, 3)$$

$$x < -1 \rightarrow -x-1 + 2x - x+2 \rightarrow y = 1$$

$$\left. \begin{aligned} \text{بیشترین} &= 3 \\ \text{کمترین} &= -1 \end{aligned} \right\} 3 - (-1) = 4$$

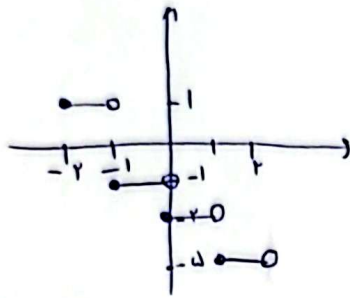
$$y = \left| \frac{1}{x} + 2 \right| - 2 + 1 \rightarrow y' = \left| \frac{1}{x} + 2 \right| - 1 = y$$

$$|x+2| - |x| = 2$$



(د)

$$f(x) = \lfloor 1-x \rfloor + \lfloor x+1 \rfloor$$



$$R = \{x | x \in \mathbb{Z}, -4 \leq x \leq 4\}$$

$$R = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

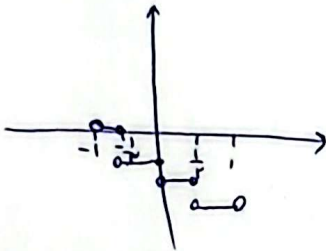
$$f(x) = \frac{1}{x} - \left\lfloor \frac{x+1}{x} \right\rfloor = \frac{1}{x} - \lfloor 1 + \frac{1}{x} \rfloor$$

$$y = \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor - 1$$

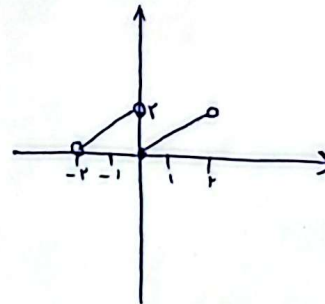
$$R = [-1, 0)$$

✓

$$y = \lfloor -x \rfloor - 1 \quad x \in (-1, 1)$$

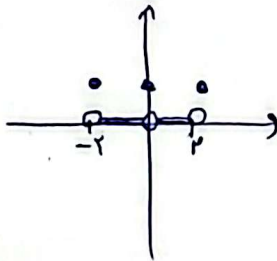


$$y = x - 2 \left\lfloor \frac{x}{2} \right\rfloor \quad x \in (-2, 2)$$

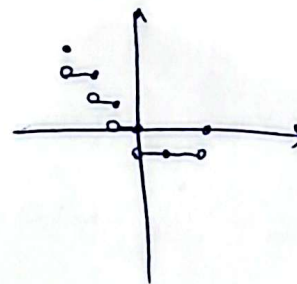


✓

$$y = \lfloor |x| - 1 \rfloor \quad x \in [-2, 2]$$



$$y = \lfloor x^2 - 2x \rfloor \quad x \in [-1, 2]$$



✓

$$a + \lfloor b \rfloor = 6/7 \rightarrow a = 1/7$$

$$b - \lfloor a \rfloor = 7/6 \rightarrow b = 7/6$$

$$\rightarrow a + \left\lfloor \frac{b}{7} \right\rfloor = 6/7 \rightarrow a = 1/7$$

$$b = 7/6$$

$$\begin{aligned} \lfloor a \rfloor + \lfloor b \rfloor &= 6 \\ \lfloor b \rfloor - \lfloor a \rfloor &= 7 \end{aligned}$$

$$\frac{2\lfloor b \rfloor}{2} = \frac{13}{2} \rightarrow \lfloor b \rfloor = 7$$

$$a + b = 1/7 + 7/6 = 6/7$$

✓

$$(1 + \sqrt{5})^4 = 194 - (1 - \sqrt{5})^4 \xrightarrow{①} (1 + \sqrt{5})^4 = 194, \dots \textcircled{2}$$

$$\textcircled{1} \rightarrow 0 < -1\sqrt{5} < 1 \xrightarrow{2\sqrt{5}} 0 < (1 - \sqrt{5})^4 < 1$$

$$\textcircled{2} \rightarrow \lfloor (1 + \sqrt{5})^4 \rfloor = 194$$

✓