

$$\frac{1}{n-1} = |n-1| \Rightarrow \frac{1}{n-1} = n-1 \Rightarrow \frac{1}{n-1} = n^2 - 2n + 1$$

$$\Rightarrow n^2 - 2n + 1 = 0 \Rightarrow n(n-2) = 0 \Rightarrow n = 0 \text{ یا } n = 2$$

$$\frac{1}{n-1} = -n+1 \Rightarrow 1 = -n^2 + 2n - 1 \Rightarrow n^2 - 2n + 2 = 0 \rightarrow \Delta < 0 \text{ ندارد}$$

معادله را در ۲ داریم

الف) $|\frac{n-1}{n}| < 2$

$$-2 < \frac{n-1}{n} < 2 \Rightarrow \frac{n-1}{n} < 2 \Rightarrow \frac{n-1-2n}{n} < 0$$

$$\frac{-1-n}{n} < 0 \Rightarrow n > 0$$

$$\frac{n-1}{n} > -2 \Rightarrow \frac{n-1+2n}{n} > 0 \Rightarrow \frac{3n-1}{n} > 0 \Rightarrow \frac{3}{n} > \frac{1}{n} \Rightarrow n > \frac{1}{2}$$

ب) $|\frac{n-2}{n+1}| > 1$

$$\frac{n-2}{n+1} > 1 \text{ یا } \frac{n-2}{n+1} < -1$$

$$\frac{n-2}{n+1} > 1 \Rightarrow \frac{n-2-n-1}{n+1} > 0 \Rightarrow \frac{-2n-3}{n+1} > 0$$

$$\frac{n-2}{n+1} < -1 \Rightarrow \frac{n-2+n+1}{n+1} < 0 \Rightarrow \frac{2n-1}{n+1} < 0$$

$$\frac{-2n-3}{n+1} > 0 \Rightarrow n < -\frac{3}{2} \text{ یا } n < -1$$

$$n^2 < |n+4| \Rightarrow n+4 > n^2 \Rightarrow n^2 - n - 4 < 0 \Rightarrow (n-3)(n+4) < 0 \Rightarrow (-4, 3)$$

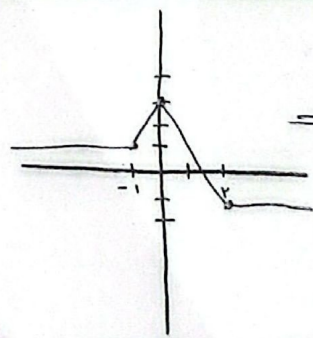
$$n+4 < -n^2 \Rightarrow n^2 + n + 4 < 0 \Rightarrow \Delta < 0 \text{ ندارد}$$

$$\Rightarrow -2 < n < 3 \Rightarrow \frac{1}{n} < -\frac{1}{2} < \frac{1}{n} \Rightarrow |n - \frac{1}{n}| < \frac{5}{2}$$

$$a = \frac{1}{n}$$

۲. $|n+1| - |2n| + |n-2|$

$-n$	-1	0	2
$ 1+2n-n+2 $	$ n+1+2n-2n $	$ n+1-2n-n+2 $	$ n+1-2n+n-2 $
$+1$	$2n+1$	$-2n+3$	-1



$$\min = -1$$

$$\max = 1$$

$$[2, 2]$$

۳. $y = |\frac{1}{n}| - 2 = \frac{1}{n} |n| - 2 \xrightarrow{\text{در دو صورت}} \frac{1}{n} |n+2| - 2 \xrightarrow{\text{در دو صورت}} y = |\frac{1}{n+2}| - 1$

$$|\frac{1}{n}| - 2 = |\frac{1}{n+2}| - 1 \Rightarrow |\frac{1}{n}| = |\frac{1}{n+2}| + 1$$

$$\frac{1}{n} = |\frac{1}{n+2}| + 1 \Rightarrow \frac{1}{n} - 1 > |\frac{1}{n+2}| \Rightarrow \frac{1}{n} - 1 < \frac{1}{n+2}$$

$$\frac{1}{n} - 1 < \frac{1}{n+2} \Rightarrow \frac{1}{n} < \frac{1}{n+2} + 1 \Rightarrow \frac{1}{n} < \frac{1}{n+2} + \frac{n+2}{n+2} \Rightarrow \frac{1}{n} < \frac{n+3}{n+2} \Rightarrow n < -3$$

الف) $f(n) = \left[\frac{1}{n} - \frac{1}{n+1} \right]$

$$\frac{1}{n} - \frac{1}{n+1} = \frac{1}{n(n+1)}$$

$R_f: \{ \omega, \varepsilon, \mu, \dots, -\omega \}$

ب) $f(n) = \frac{1}{n} - \left[\frac{n+1}{n} \right]$

$$\frac{1}{n} - \left[1 + \frac{1}{n} \right] = \frac{1}{n} - \left[\frac{n+1}{n} \right] = -1$$

$R_f: [-1, 0)$

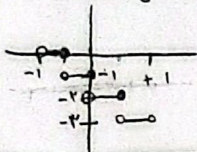
الف) $y = \lfloor -2n \rfloor - 1$ $n \in (-1, 1) \Rightarrow \lfloor -2n \rfloor \in \{-2, -1, 0\}$

$-2 < -2n < -1 \Rightarrow \lfloor -2n \rfloor = -2 \Rightarrow y = -3, -1 < n < -0.5$

$-1 < -2n < 0 \Rightarrow \lfloor -2n \rfloor = -1 \Rightarrow y = -2, -0.5 < n < 0$

$0 < -2n < 1 \Rightarrow \lfloor -2n \rfloor = 0 \Rightarrow y = -1, 0 < n < 0.5$

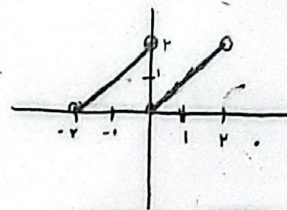
$1 < -2n < 2 \Rightarrow \lfloor -2n \rfloor = 1 \Rightarrow y = 0, 0.5 < n < 1$



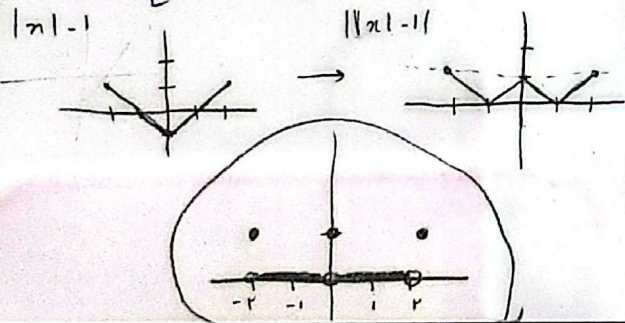
ب) $y = n - \lfloor \frac{n}{r} \rfloor$ $n \in (-r, r) \Rightarrow \lfloor \frac{n}{r} \rfloor \in \{-1, 0, 1\}$

$-1 < \frac{n}{r} < 0 \Rightarrow \lfloor \frac{n}{r} \rfloor = -1 \Rightarrow y = n + r, -r < n < 0$

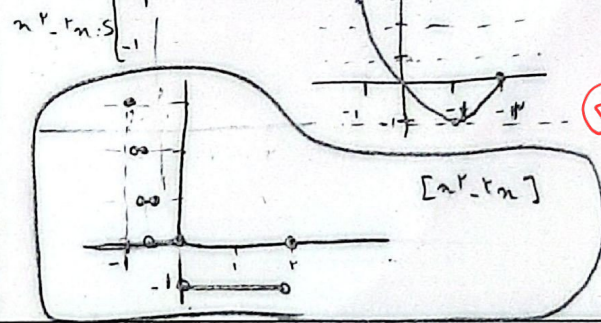
$0 \leq \frac{n}{r} < 1 \Rightarrow \lfloor \frac{n}{r} \rfloor = 0 \Rightarrow y = n, 0 \leq n < r$



الف) $y = \lfloor |n| - 1 \rfloor$ $n \in [-r, r]$



ب) $y = \lfloor n \rfloor - \lfloor rn \rfloor$ $n \in [-1, r]$



$a + \lfloor b \rfloor \geq \varepsilon, \delta \quad b - \lfloor a \rfloor \geq \varepsilon, \delta \quad a + b$

$\lfloor a \rfloor + \lfloor b \rfloor \geq \varepsilon, \delta \quad \lfloor b \rfloor - \lfloor a \rfloor \geq \varepsilon, \delta$

$\lfloor b \rfloor = 3, \lfloor a \rfloor = 1$

$b - 1 = \varepsilon, \delta \Rightarrow b \geq 1 + \varepsilon, \delta$

$a + 3 \geq \varepsilon, \delta \Rightarrow a \geq \varepsilon, \delta - 3$

$\Rightarrow a + b \geq \varepsilon, \delta - 3 + 1 + \varepsilon, \delta = \varepsilon, \delta - 2 + \varepsilon, \delta = \varepsilon, \delta$

$(1 + \sqrt{2})^4 + (1 - \sqrt{2})^4 = 198$

$(1 + \sqrt{2})^4 = 198 - (1 - \sqrt{2})^4$

$\lfloor (1 + \sqrt{2})^4 \rfloor = \lfloor 198 - (1 - \sqrt{2})^4 \rfloor = \lfloor 198 - \underbrace{(1 - \sqrt{2})^4}_{\approx 0.07} \rfloor = 197$

$\Rightarrow \lfloor (1 + \sqrt{2})^4 \rfloor = 197$

$-1 < (1 - \sqrt{2})^4 < 0$