

الف) $f(n) = [1/n - 1/n + \epsilon]$

$$\frac{1-n+n+\epsilon}{\omega} = \frac{1-n-n-\epsilon}{-1/n-\omega} = \frac{n-1-\epsilon}{-1/n-\omega}$$

$R_f: \{\omega, \epsilon, \omega, \dots, -\omega\}$

ب) $f(n) = \frac{1}{n} - \left[\frac{n+1}{n} \right]$

$$\frac{1}{n} - \left[1 + \frac{1}{n} \right] = \frac{1}{n} - \left[\frac{1}{n} \right] - 1$$

$0 < \frac{1}{n} < 1$

$R_f: [-1, 0)$

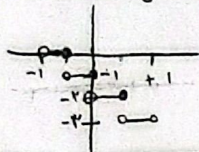
الف) $y = [2n] - 1$ $n \in (-1, 1) \Rightarrow -2 < 2n < 2$

$-2 < 2n < -1 \Rightarrow [2n] = -2 \Rightarrow y = -2 - 1 = -3$

$-1 < 2n < 0 \Rightarrow [2n] = -1 \Rightarrow y = -1 - 1 = -2$

$0 < 2n < 1 \Rightarrow [2n] = 0 \Rightarrow y = 0 - 1 = -1$

$1 < 2n < 2 \Rightarrow [2n] = 1 \Rightarrow y = 1 - 1 = 0$



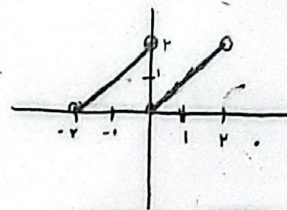
ب) $y = n - r \left[\frac{n}{r} \right]$ $n \in (-r, r) \Rightarrow -r < n < r$

$-r < n < -r/2 \Rightarrow \left[\frac{n}{r} \right] = -1 \Rightarrow y = n + r$

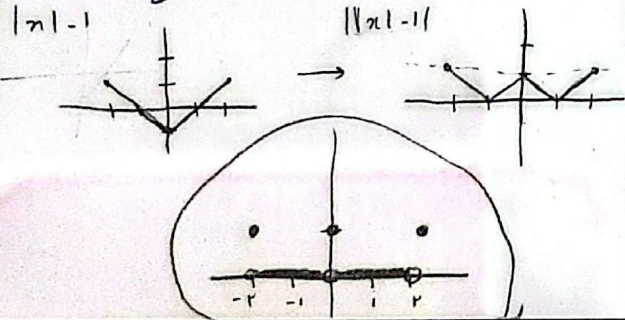
$-r/2 < n < 0 \Rightarrow \left[\frac{n}{r} \right] = 0 \Rightarrow y = n$

$0 < n < r/2 \Rightarrow \left[\frac{n}{r} \right] = 0 \Rightarrow y = n$

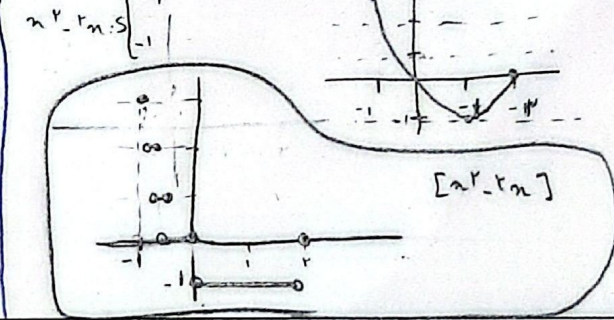
$r/2 < n < r \Rightarrow \left[\frac{n}{r} \right] = 1 \Rightarrow y = n - r$



الف) $y = [1/n - 1]$ $n \in [-2, 2]$



ب) $y = [n/r - r]$ $n \in [-1, 2]$



$a + [b] \geq 2, 2$ $b - [a] \geq 2, 2$ $a + b$

$[a] + [b] \geq 2$ $[b] - [a] \geq 2$

$[b] = 3, [a] = 1$

$b - 1 = 2, 2 \Rightarrow b \geq 3, 2$

$a + 3 \geq 2, 2 \Rightarrow a \geq 1, 2$ $\Rightarrow a + b \geq 3, 2 + 1, 2 \Rightarrow [3, 4]$

$(1 + \sqrt{2})^4 + (1 - \sqrt{2})^4 = 198$

$[(1 + \sqrt{2})^4]$

$(1 + \sqrt{2})^4 = 198 - (1 - \sqrt{2})^4$

$[(1 + \sqrt{2})^4] = [198 - (1 - \sqrt{2})^4] = [198 - (1 - \sqrt{2})^4]$

$0 < (1 - \sqrt{2})^4 < 1 \Rightarrow [198 - (1 - \sqrt{2})^4] = 198 - 1 = 197$

$-1 < -(1 - \sqrt{2})^4 < 0$