

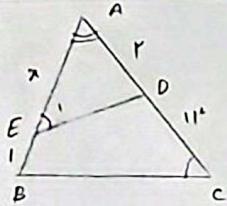
$$\frac{ED}{AE} = 1 \Rightarrow ED = 1-x, AE = x \Rightarrow \frac{ED}{AE} = 1$$

$$\Rightarrow BE^2 = 1^2 + x^2 = 1 + x^2 \Rightarrow BE = \sqrt{1+x^2}$$

$$AC^2 = 1^2 + 1^2 = 2 \Rightarrow AC = \sqrt{2}$$

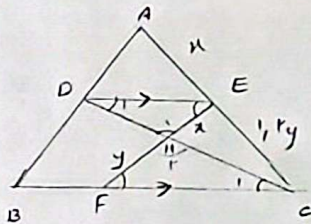
$$BC \parallel AE \Rightarrow \hat{C}_1 = \hat{A}_1, \hat{F}_1 = \hat{F}_2 \Rightarrow \triangle BCF \sim \triangle AEF$$

$$\Rightarrow \frac{BF}{EF} = \frac{CF}{AF} \Rightarrow \frac{BF+CF}{EF} = \frac{CF+AF}{AF} \Rightarrow \frac{BC}{EF} = \frac{AC}{AF} \Rightarrow \frac{EF}{AF} = \frac{BE}{AC} = \frac{\sqrt{1+x^2}}{\sqrt{2}}$$



$$\hat{A}_1 = \hat{A}_2 \Rightarrow \triangle AED \sim \triangle ABC \Rightarrow \frac{AD}{AB} = \frac{AE}{AC} \Rightarrow \frac{x}{1} = \frac{AE}{1} \Rightarrow AE = x$$

$$\Rightarrow x^2 + x = 1 \Rightarrow x^2 + x - 1 = 0 \Rightarrow (x+4)(x-1) = 0 \Rightarrow x = -4 \text{ or } x = 1$$

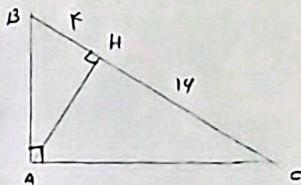


$$DE \parallel BC \Rightarrow \frac{AE}{AC} = \frac{DE}{BC} \Rightarrow \frac{1}{1} = \frac{DE}{1} \Rightarrow DE = 1$$

$$\Rightarrow \frac{AE}{AC} = \frac{x}{1} = \frac{x}{1} = \frac{DE}{BC} \Rightarrow DE = x, BC = 1$$

$$DE \parallel FC \Rightarrow \hat{D}_1 = \hat{C}_1, \hat{H}_1 = \hat{H}_2 \Rightarrow \triangle DEH \sim \triangle FHC \Rightarrow \frac{EH}{FH} = \frac{DE}{FC} = \frac{x}{1} = \frac{1}{1} = \frac{a}{FC}$$

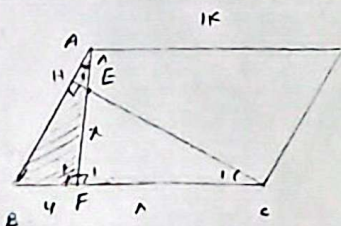
$$\Rightarrow FC = \frac{a}{1} \Rightarrow BF = \frac{9a}{1} - \frac{a}{1} = \frac{8a}{1} = 8 \Rightarrow a = \frac{8}{1} \Rightarrow BC = \frac{14}{1} = 14$$



$$AB^2 = 1^2 + 14^2 = 197 \Rightarrow AB = \sqrt{197}$$

$$AC^2 = 14^2 + 1^2 = 197 \Rightarrow AC = \sqrt{197}$$

$$\Rightarrow \frac{AB}{AC} = \frac{\sqrt{197}}{\sqrt{197}} = 1$$

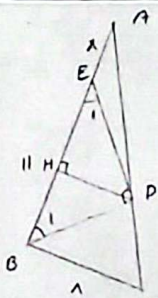


$$\triangle ABF, \triangle CBH \rightarrow \hat{A}_1 = \hat{C}_1$$

$$\triangle ABF, \triangle EFC : \hat{F}_1 = \hat{F}_2 = 1 \Rightarrow \triangle ABF \sim \triangle EFC$$

$$\Rightarrow \frac{AF}{FC} = \frac{BF}{EF} \Rightarrow \frac{1+x}{1} = \frac{4}{x} \Rightarrow 1+x = \frac{4}{x} \Rightarrow x^2 + x = 4 \Rightarrow x^2 + x - 4 = 0 \Rightarrow (x+4)(x-1) = 0 \Rightarrow x = -4 \text{ or } x = 1$$

$$\rightarrow AF = AE + EF = 1 + 1 = 2$$



$$\frac{AD}{DC} = \frac{AB}{BC} = \frac{x+11}{14}$$

نسبت اضلاع متساوی الساقین

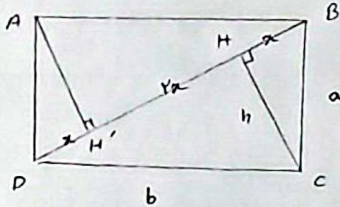
$$\rightarrow DH \perp AB \Rightarrow AB \perp BC \Rightarrow \hat{B} = 90^\circ \xrightarrow{\text{نسبت}} \hat{B}_1 = 90^\circ$$

$$\rightarrow \triangle BED : \hat{D} = 90^\circ, \hat{B}_1 = 90^\circ \Rightarrow \hat{E}_1 = 90^\circ \Rightarrow ED = BD = a \rightarrow \frac{a}{14} = \frac{11}{14} \Rightarrow a = \frac{11}{\sqrt{2}}$$

$$\rightarrow BD \times ED = HD \times 11 \Rightarrow \frac{11}{\sqrt{2}} \times \frac{11}{\sqrt{2}} = 11 \times HD \rightarrow HD = \frac{11}{2}$$

$$\frac{AD}{AC} = \frac{HD}{BC} = \frac{\frac{11}{2}}{14} = \frac{11}{28} = \frac{x+11}{14} = \frac{x+11}{14} \Rightarrow 10.9 + 11x = 14x + 14 \rightarrow 4x = 4.9 \Rightarrow x = \frac{4.9}{4}$$

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$$a^r = HB \times BD = x \times \sqrt{2}x = \sqrt{2}x^r \rightarrow a = \sqrt{2}x$$

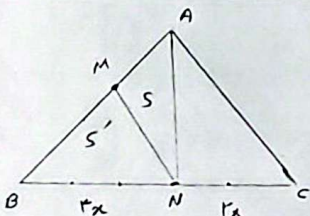
$$b^r = DH \times BD = \sqrt{2}x \times \sqrt{2}x = 2x^r \rightarrow \sqrt{2}x = b$$

$$h^r = HB \times DH = x \times \sqrt{2}x = \sqrt{2}x^r \rightarrow h = \sqrt{2}x$$

$$S_{\triangle BEH} = \frac{x \times \sqrt{2}x}{2} = \frac{\sqrt{2}x^2}{2}$$

$$S_{ABCD} = ab = \sqrt{2}x \times \sqrt{2}x = 2x^2 \rightarrow \frac{\sqrt{2}x^2}{2} = \frac{1}{2} \times 2x^2 = x^2$$

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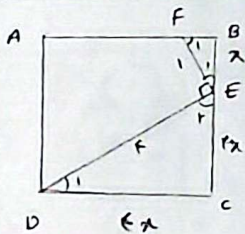
$$S_{AMN} = S, S_{BMN} = S'$$

$$\rightarrow \frac{S_{ACN}}{S_{ABN}} = \frac{r}{r} = \frac{S_{ACN}}{S+S'} \rightarrow S_{ACN} = \frac{r(S+S')}{r}$$

$$\rightarrow S_{ABC} = \frac{r}{r}(S+S') + (S+S') = \frac{2(S+S')}{r}$$

$$\rightarrow \frac{S_{ABC}}{S_{BMN}} = r = \frac{2(S+S')}{r} = \frac{2(S+S')}{r} \Rightarrow 2S' = 2S + 2S' + 2S' = 4S' \Rightarrow \frac{S'}{S} = \frac{2}{r} = \frac{BM}{AM}$$

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$$\hat{E}_1 + \hat{E}_r = 90^\circ \Rightarrow \hat{E}_r = \hat{F}_1$$

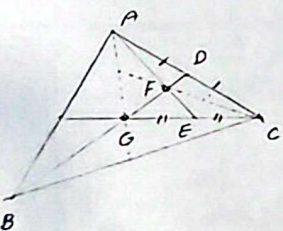
$$\hat{E}_1 + \hat{F}_1 = 90^\circ$$

$$\hat{B} = \hat{C} = 90^\circ \Rightarrow \triangle FEB \sim \triangle CDE \Rightarrow \frac{BE}{CD} = \frac{FE}{DE} = \frac{1}{r} \rightarrow BE = x, CD = \sqrt{2}x$$

$$\rightarrow BC = CD = \sqrt{2}x = BE + EC \rightarrow EC = \sqrt{2}x$$

$$2x^2 + 14x^2 = 2\sqrt{2}x^2 = 14 \rightarrow x^2 = \frac{14}{2\sqrt{2}} \rightarrow x = \frac{\sqrt{2}}{2} \rightarrow (F_x)^2 = \left(\frac{14}{2}\right)^2 = \frac{196}{4}$$

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$$\frac{GD}{BD} = \frac{1}{r} \rightarrow BD = rGD$$

G محل برخورد میانخطی مثلث ABC

$$\frac{FD}{GD} = \frac{1}{r} \Rightarrow FD = \frac{GD}{r}$$

F محل برخورد میانخطی مثلث AGC

$$\rightarrow \frac{BD}{FD} = \frac{rGD}{\frac{GD}{r}} = r^2 = 9$$

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