حسین مرادی

۱۳٫۷۵

1)

$3^{Ax+B} = x^2 \xrightarrow{x=1} 3^{A+B} = 1^2 = 1 \Rightarrow A+B=0$

$3^{3A+B} = 9 = 3^2 \Rightarrow 3A+B=2$

$B=-A \Rightarrow 3A-A=2 \quad 2A=2 \quad A=1$

$f(0) = 3^B = 3^{-1} = \frac{1}{3} \qquad B=-1$

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2)

$4^x + 15 = 2^{x+3}$

$4^x = 2^{2x} \Rightarrow 2^{2x}+15 = 2^{x+3}$

$t = 2^x \Rightarrow t^2 + 15 = 8t$

$t^2 - 8t + 15 = 0 \quad (t-3)(t-5)=0$

$2^x = 3 \Rightarrow x = \log_2^3$

$2^x = 5 \Rightarrow x = \log_2^5$

$\log_2^3 + \log_2^5 = \log_2^{15}$

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3)

$\log_{21}^{147} = \log_{21}^{(3\times 7^2)} = \log_{21}^3 + 2\log_{21}^7$

$\log_{21}^{1323} = \log_{21}^{(3^3\times 7^2)} = 3\log_{21}^3 + 2\log_{21}^7$

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$(\log_{21}^3 + 2\log_{21}^7)(3\log_{21}^3 + 2\log_{21}^7) =$

$4(\log_{21}^3)^2 + 8\log_{21}^3 \; 2\log_{21}^7 + 4(\log_{21}^7)^2$

$4(\log_{21}^3 + \log_{21}^7)^2 = 4(\log_{21}^{21})^2 = 4$

4)

$x^2 - 2x + 1 = (x-1)^2$

$\log((x-1)^2) + 3\log(1-x) =$ ۱٫۷۵

$2\log|x-1| + 3\log(1-x)$

$|x-1| = 1-x \Rightarrow 1-x>0 \Rightarrow x<1$

$2\log(1-x) + 3\log(1-x) = 5$

$5\log(1-x) = 5 \quad \log(1-x) = 1$

$1-x = 10 \quad x = -9$

$\log(-x) = \log 9$

$\log_3^{-9} = 2$

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۵) $(x-2)(x^2+2x+4) = (x^3-8) = 8 \to x = \sqrt[3]{16}$

$\log_{\sqrt[3]{2}} x = 4$

۶) $\log((2-x)\times(x-2)^2) = 3$

$(x-2)^2 = (2-x)^2$

$(2-x)\times(2-x)^2 \Rightarrow \log(2-x)^3 = 3$

$(2-x)^3 = 10^3 \Rightarrow 2-x = 10 \quad x = -8$

$\log_{\sqrt{2}}(-x) = \log_{\sqrt{2}} 8$

$8 = 2^3 , \sqrt{2} = 2^{\frac{1}{2}} \Rightarrow \log_{\sqrt{2}} 8 = \frac{3}{\frac{1}{2}} = 6$

۷) $x^2-4x-2=0 \to x_1 = 2+\sqrt{6}$ ✓, $x_2 = 2-\sqrt{6}$ ✗

$\log_6 (x-2) = \log_6 \sqrt{6} = \frac{1}{2}$

۸) $\log_{18} 8 = \frac{\log_3 8}{\log_3 18} \to 3\log_3 2 = \frac{15}{8}$

$\log_3 18 \to \log_3 2 + 2 = \frac{5}{8} + 2 = \frac{21}{8}$

$\frac{\frac{15}{8}}{\frac{21}{8}} = \frac{15}{21} = \frac{5}{7}$

۹) $\log_{12} 6 = \frac{\log_4 6}{\log_4 12} \to \log_4 2 + \log_4 3 = \log_4 6$

$\log_4 2 = \frac{1}{2}$

$\log_4 6 = \frac{1}{2} + 0.8 = 1.3$

$\log_4 12 = 0.8 + 1 = 1.8 \qquad \log_{12} 6 = \frac{1.3}{1.8} = \frac{13}{18}$

۱۰) $(a\log 2) - a + b\log 2 = 0 \to a(1-\log 2) = b\log 2$

$\to a\log 5 = b\log 2 \to \frac{b}{a} = \log_2 5 \to (\sqrt{2})^{\log_2 5} = \sqrt{5}$