آرتیست اکبری — تکلیف (۲٤) — یازدهم دختر B

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سوال ①

$x=1 \to y=x^2 \to (1,1) \Rightarrow 3^{A+B} = 1 \to$

$x=3 \to y=x^2 \to (3,9) \Rightarrow 3^{3A+B} = 9 \to$

$\begin{cases} A+B=0 \\ 3A+B=2 \end{cases}$

$A=1 , B=-1$

$f(x) = 3^{x-1}$

نقطه تقاطع با محور y $\to 3^{0-1} = y \to \boxed{y=\frac{1}{3}}$

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سوال ②

$\log_2^{4^x+15} = x+3$

$4^x+15 = 2^{x+3} \to 2^{2x} - 2^{x+3} + 15 = 0 \to (2^x)^2 - 8\times 2^x + 15 = 0 \xrightarrow{2^x=t} t^2 - 8t + 15 = 0 \to (t-3)(t-5)=0$

$t = 3 , 5$

$t=3 \to 2^x = 3 \to \boxed{x=\log_2^3}$

$t=5 \to 2^x = 5 \to \boxed{x=\log_2^5}$

$\log_2^3 + \log_2^5 = \log_2^{15}$

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سوال ③

$(\log_{21}^3)^2 , \log_{21}^{147} \log_{21}^{1323}$

$(\log_{21}^3)^2 + (\log_{21}^7 + \log_{21}^{21})(\log_{21}^3 + 2\log_{21}^{21}) \to (\log_{21}^3)^2 + (2 - \log_{21}^3)(2 + \log_{21}^3)$

$\log_{21}^{\frac{21}{3}} = \log_{21}^{21} - \log_{21}^3$

$= (\log_{21}^3)^2 - (\log_{21}^3)^2 + 4 = \boxed{4}$

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سوال ④

$\log(x^2-2x+1) + 3\log(1-x) = 5 , \log_3^{(-x)} = ?$

$\log^{(1-x)^2} + \log^{(1-x)^3} = 5 \to (1-x)^5 = 10^5 \to 1-x = 10 \to x = -9$

$\log_3^{-x} \to \log_3^9 = \boxed{2}$

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سوال ⑤

$\log_2^{(x^2+2x+4)} + \log_2^{(x-2)} = 3 , \log_{\sqrt{2}}^x$

$\log_2^{(x^2+2x+4)(x-2)} = 3 \to \log_2^{x^3-8} = 3 \to x^3 - 8 = 8 \to x = \sqrt[3]{16}$

$\log_{\sqrt{2}}^x \to \log_{\sqrt{2}}^{\sqrt[3]{16}} = \log_{2^{\frac{1}{3}}}^{2^{\frac{4}{3}}} \to \frac{\frac{4}{3}}{\frac{1}{3}} \log_2^2 = \boxed{4}$

سوال ۶

$\log(2-x) - \log\frac{1}{(x-2)^2} = 4 \quad , \log_{\sqrt{2}}^{(-x)}$

$\log(2-x) - \log(x-2)^{-2} = 4$

$3\log(2-x) = 4 \longrightarrow \log(2-x) = 1 \rightarrow 2-x = 10 \rightarrow x = -8$

$\log_{\sqrt{2}}^{(-x)} \rightarrow \log_{\sqrt{2}}^{8} = \log_2^{\frac{1}{2}\times 3} = \boxed{6}$

سوال ۷

$x^{x^2-2x} = 16^x \quad , \log_4^{(x-2)} = ?$

$x^{x^2-2x} = 16^x \longrightarrow x^2-2x = 4x \rightarrow x^2-6x = 0 \xrightarrow{\text{مرتبع کامل}} (x-3)^2 - 9 = 0 \rightarrow \underbrace{(x-3)^2 = 9}$

$x-3 = 3 \rightarrow x = 6$

$x-3 = -3 \rightarrow x = 0$ ق ق ت

$\log_4^{x-2} \longrightarrow \log_4^{6-2} = \log_4^4 = \boxed{\frac{1}{2}}$

سوال ۸

$\log_3^2 = \frac{a}{b} \quad , \log_{12}^{8} = ?$

$\log_3^2 = \frac{a}{b} \rightarrow \log_2^3 = \frac{b}{a}$

$\log_{12}^{8} = \frac{1}{\log_8^{12}} \longrightarrow \log_8^{12} = \log_{2^3}^{3\times 2^2} \rightarrow \log_{2^3}^{2^2} + \log_{2^3}^{3} = \frac{1}{3}\log_2^2 + \frac{1}{3}\log_2^3 \rightarrow \frac{2}{3} + \frac{1}{3}\times\frac{b}{a} = \frac{2a+b}{3a}$

$\log_{12}^{8} = \frac{1}{\frac{2a+b}{3a}} = \frac{3a}{2a+b} = \boxed{\frac{6}{7}}$

سوال ۹

$\log_3^4 = 0/7 \quad , \log_{12}^{6} = ?$

$\log_{12}^{6} = \frac{\log_4^6}{\log_4^{12}} = \frac{\log_2^2 + \log_4^3}{\log_4^3 + \log_4^4} = \frac{0/5 + 0/7}{0/7 + 1} = \frac{1/2}{1/7} = \boxed{\frac{12}{17}}$

سوال ۱۰

$(a\log 2)x^2 + ax + b\log 2 = 0 \quad , x = -1 \text{ ریشه } (\sqrt{2})^{\frac{b}{a}} = ?$

$x = -1 \longrightarrow a\log 2 - a + b\log 2 = 0 \xrightarrow{\log 2 = t} at - a + bt = 0 \xrightarrow{\div a} t - 1 + \frac{b}{a}t = 0$

$t(1+\frac{b}{a}) = 1 \rightarrow t = \frac{1}{1+\frac{b}{a}} \longrightarrow \log_{10}^2 = \frac{1}{1+\frac{b}{a}} \longrightarrow \log_2^{10} = 1 + \frac{b}{a} \rightarrow \log_2^2 + \log_2^5 = 1 + \frac{b}{a}$

$\log_2^5 = \frac{b}{a}$

$(\sqrt{2})^{\frac{b}{a}} \longrightarrow \sqrt{2}^{\log_2^5} \longrightarrow 5^{\log_2^{\sqrt{2}}} = 5^{\frac{1}{2}} = \boxed{\sqrt{5}}$