سوال ۱

$x=1 \longrightarrow y=x^2 \longrightarrow (1,1) \Rightarrow 3^{A+B}=1 \longrightarrow A+B=0$

$x=2 \longrightarrow y=x^2 \longrightarrow (2,4) \Rightarrow 3^{2A+B}=9 \longrightarrow 2A+B=2$

$A=1 \ , \ B=-1$

$f(x)=3^{x-1}$

نقطه تلاقی با محور y ها $\longrightarrow 3^{0-1}=y \longrightarrow \boxed{y=\frac{1}{3}}$

سوال ۲

$\log_2^{4x+1\omega} = x+2$

$4^x+1\omega=2^{x+2} \longrightarrow 2^{2x}-2^{x+2}+1\omega=0 \rightarrow (2^x)^2-4\times2^x+1\omega=0 \xrightarrow{2^x=t} t^2-4t+1\omega=0 \rightarrow (t-2)(t-\omega)=0$

$t=2,\omega$

$t=2 \longrightarrow 2^x=2 \longrightarrow \boxed{x=\log_2^2}$

$t=\omega \longrightarrow 2^x=\omega \longrightarrow \boxed{x=\log_2^\omega}$

سوال ۳

$(\log_{21}^3)^2+\log_{21}^{147}\log_{21}^{343}$

$(\log_{21}^3)^2+(\log_{21}^7+\log_{21}^3)(\log_{21}^3+2\log_{21}^7) \longrightarrow (\log_{21}^3)^2+(2+\log_{21}^3)(2\log_{21}^7+\log_{21}^3)$

$\log_{21}^{21}=\log_{21}^{21}-\log_{21}^3$

$=(\log_{21}^3)^2-(\log_{21}^3)^2+4=\boxed{4}$

سوال ۴

$\log(x^2-x+1)+2\log(1-x)=\omega \ , \ \log_3^{(-x)}=?$

$\log^{(1-x)^2}+\log^{(1-x)^3}=\omega \rightarrow (1-x)^\omega=10^\omega \rightarrow 1-x=10 \longleftrightarrow x=-9$

$\log_3^{-x} \rightarrow \log_3^9=\boxed{2}$

سوال ۵

$\log_2^{(x^2+2x+4)}+\log_2^{(x-2)}=3 \ , \ \log_{\sqrt[3]{2^{12}}}^x$

$\log_2^{(2x^2+2x+4)(x-2)}=3 \rightarrow \log_2^{x^3-1}=3 \rightarrow x^3-1=8 \longrightarrow x=\sqrt[3]{16}$

$\log_{\sqrt[3]{2}}^x \longrightarrow \log_{\sqrt[3]{2}}^{\sqrt[3]{16}}=\log_{2^{\frac{1}{3}}}^{2^{\frac{4}{3}}} \longrightarrow \frac{\frac{4}{3}}{\frac{1}{3}(2)}\log_2^2=\boxed{4}$

سوال ۶

$\log(2-x) - \log\frac{1}{(x-2)^2} = 2 \quad ; \log_{\sqrt{2}}^{(-x)}$

$\log(2-x) - \log(x-2)^{-2} = 2$

$3\log(2-x) = 3 \longrightarrow \log(2-x) = 1 \rightarrow 2-x = 10 \rightarrow x = -8$

$\log_{\sqrt{2}}^{(-x)} \rightarrow \log_{\sqrt{2}}^{8} = \log_2^{\frac{3}{2}} = \boxed{6}$

سوال ۷

$x^{x^2-2x} = 16^x \quad , \log_6^{(x-2)} = ?$

$x^{x^2-2x} = 16^x \longrightarrow x^2-2x = 4x \rightarrow x^2-6x = 0 \xrightarrow{\text{مرتبع کامل}} (x-3)^2 - 9 = 0 \rightarrow (x-3)^2 = 9$

$x-3 = 3 \rightarrow x = 6$

$x-3 = -3 \rightarrow x = 0$ ق ق ن

$\log_6^{x-2} \longrightarrow \log_6^{6-2} = \log_6^4 = \boxed{\frac{2}{3}}$

سوال ۸

$\log_3^2 = \frac{a}{b} \quad , \log_{12}^8 = ?$

$\log_3^2 = \frac{a}{b} \rightarrow \log_2^3 = \frac{b}{a}$

$\log_{12}^8 = \frac{1}{\log_8^{12}} \longrightarrow \log_8^{12} = \log_{2^3}^{3\times 2^2} \rightarrow \log_{2^3}^{2^2} + \log_{2^3}^{3} = \frac{2}{3}\log_2^2 + \frac{1}{3}\log_2^3 \rightarrow \frac{2}{3} + \frac{1}{3}\times\frac{5}{3} = \frac{11}{9}$

$\log_{12}^8 = \frac{1}{\frac{11}{9}} = \frac{9}{11} = \boxed{\frac{9}{11}}$

سوال ۹

$\log_3^4 = 0/1 \quad , \log_{12}^6 = ?$

$\log_{12}^6 = \frac{\log_4^6}{\log_4^{12}} = \frac{\log_2^2 + \log_2^3}{\log_2^4 + \log_2^3} = \frac{0/3 + 0/5}{2 + 0/5} = \frac{1/3}{1/1} = \boxed{\frac{12}{11}}$

سوال ۱۰

$(a\log 2)x^2 + ax + b\log 2 = 0 \quad , x = -1$ ریشه $(\sqrt{2})^{\frac{b}{a}} = ?$

$x = -1 \longrightarrow a\log 2 - a + b\log 2 = 0 \xrightarrow{\log 2 = t} at - a + bt = 0 \xrightarrow{\div a} t - 1 + \frac{b}{a}t = 0$

$t(1+\frac{b}{a}) = 1 \rightarrow t = \frac{1}{1+\frac{b}{a}} \longrightarrow \log_{10}^2 = \frac{1}{1+\frac{b}{a}} \longrightarrow \log_2^{10} = 1 + \frac{b}{a} \rightarrow \log_2^2 + \log_2^5 = 1 + \frac{b}{a}$

$\frac{b}{a} = \log_2^5$

$(\sqrt{2})^{\frac{b}{a}} \longrightarrow \sqrt{2}^{\log_2^5} \longrightarrow 5^{\log_2^{\sqrt{2}}} = 5^{\frac{1}{2}} = \boxed{\sqrt{5}}$