① $y = x^2$: $x=1 \to y=1$; $x=2 \to y=4$

$f(x) = 3^{Ax+B}$ ⟶ $1 = 3^{A+B}$, $4 = 3^{2A+B}$

$\begin{cases} A+B=0 \\ 2A+B=2 \end{cases} \Rightarrow A=1 \quad B=-1$

$\Rightarrow f(x) = 3^{x-1} \xrightarrow{x=0} 3^{-1} = \frac{1}{3}$

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② $\log_2^{(4^x+15)} = x+3$

$2^{2x} + 15 = 2^{x+3} = 2^x \times 8$

$2^{2x} + 15 = 8 \times 2^x \quad t = 2^x \Rightarrow t^2 - 8t + 15 = 0 \quad (t-3)(t-5)=0 \begin{cases} t=3 \\ t=5 \end{cases}$

$2^x = 3 \Rightarrow x = \log_2^3$ $\quad 2^x = 5 \Rightarrow x = \log_2^5$

$\log_2^3 + \log_2^5 = \log_2^{15}$

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③ $(\log_{21}^3)^2 + \log_{21}^{147} \times \log_{21}^{1323}$

$\log_{21}^{147} = \log_{21}^3 + 2\log_{21}^7$

$\log_{21}^3 + \log_{21}^7 = 1 \Rightarrow \log_{21}^3 = t \longrightarrow \log_{21}^7 = 1-t$

$\log_{21}^{147} = t + 2(1-t) = 2-t$

$\log_{21}^{1323} = \log_{21}^{3^3 \times 7^2} = 3t + 2(1-t) = t+2$

$\Rightarrow t^2 + (2-t)(t+2) = t^2 + 4 - t^2 = 4$

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۴) $\log(x-1)^2 + 3\log(1-x) = 5 \quad 2\log(1-x) + 3\log(1-x) = 5$

$\Rightarrow \log_{10}(1-x) = 1 \Rightarrow 1-x = 10 \quad x = -9 \quad \log_3^9 = 2$

۵)

$\log_2(x^2+2x+3) + \log_2(x-2) = 3 = \log_2^8$

$(x-2)(x^2+2x+4) = 8 \longrightarrow (x-2)^3 = 8 \Rightarrow x-2 = 2 \Rightarrow x = 4$

$\log_{\sqrt{2}}^4 = 3\log_2^4 = 3\times 2 = 6$

۶) $\log(2-x) - \log\frac{1}{(2-x)^2} = 3$

$\log(2-x) + 2\log(2-x) = 3\log(2-x) = 3 \Rightarrow \log_{10}(2-x) = 1$

$2-x = 10 \quad x = -8 \quad \log_{\sqrt{2}}^8 = \frac{3}{2}\log_2^2 = \frac{3}{2}$

۷) $3^{x^2-2} = 81^x \longrightarrow 3^{x^2-2} = 3^{4x}$

$x^2 - 4x - 2 = 0 \Rightarrow x = 2 \pm \sqrt{6}$

$\log_6^{(x-2)} \xrightarrow{x=2+\sqrt{6}} \log_6^{\sqrt{6}} = \frac{1}{2}$

۸) $\log_{18}^8 = 3\log_{18}^2 \xrightarrow{\text{تغییر مبنا}} \log_{18}^2 = \frac{\log_3^2}{\log_3^{18}} = \frac{5}{\log_3^9 + \log_3^2}$

$= \frac{5}{2+5} \rightsquigarrow 3\log_{18}^2 = 3\left(\frac{5}{2\times 5}\right) = \frac{5}{7}$

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(9)

$$\log_{12} 6 = \frac{\log_4 6}{\log_4 12} = \frac{\log_4 2 + \log_4 3}{\log_4 3 + \log_4 4} = \frac{\frac{1}{2} + \frac{4}{5}}{\frac{4}{5} + 1} = \frac{13}{18}$$

(16)

$x=-1 \longrightarrow a\log 2 - a_9 b\log 2 = 0$

$a(\log 2 - 1) + b\log 2 = 0$

$\frac{b}{a} = \frac{1-\log 2}{\log 2}$ $\qquad (\sqrt{2})^{\frac{b}{a}} = 2^{\frac{b}{2a}} = 2^{\frac{1-\log 2}{2\log 2}}$

$\longrightarrow 2 = 10^{\log 2} \Rightarrow 2^{\frac{1}{2\log 2}} = (10^{\log 2})^{\frac{1}{2\log 2}} = 10^{\frac{1}{2}} = \sqrt{10}$

$2^{\frac{1}{2\log 2} - \frac{1}{2}} = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$

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