نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره «۲۲»، کلاس یازدهم .....

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$f(x)=3^{Ax+B}$ ، $y=x^2$

x	1	3
y	1	9

$1=3^{A+B}$

$9=3^{3A+B}$

$\Rightarrow \begin{cases} A+B=0 \\ 3A+B=2 \end{cases}$

$2A=2 \Rightarrow A=1$ ، $B=-1$

$y=3^{x-1}$

$x=0 \Rightarrow y=3^{-1}=\boxed{\frac{1}{3}}$

۲

$\log_2^{(x^2+15)} = x+3$

$2^{x+3} = 2^x + 15 \Rightarrow 2^{2x} + 15 = \underbrace{2^{x+3}}_{2^x \times 2^3}$

$2^x = t$

$\Rightarrow t^2+15=8t \Rightarrow t^2-8t+15=0 \Rightarrow (t-3)(t-5)=0 \Rightarrow t=\begin{cases}3\\5\end{cases}$

$2^x=3 \Rightarrow x=\log_2^3$ ، $2^x=5 \Rightarrow x=\log_2^5$

$\log_2^5+\log_2^3=\log_2^{15} \simeq 3,91$

۳

$(\log_{21}^3)^2+\log_{21}^{175}\log_{21}^{1323}$

$\log_{21}^3=a$ ، $\log_{21}^7=b$

$a+b=\log_{21}^3+\log_{21}^7=\log_{21}^{3\times7}=1$ / $(\log_{21}^3)^2=a^2$

$\log_{21}^{175} = \log_{21}^{3\times7^2} = \log_{21}^3+2\log_{21}^7 = a+2b$

$\log_{21}^{1323} = \log_{21}^{3^3\times7^2} = 3\log_{21}^3+2\log_{21}^7 = 3a+2b$

$a^2+(a+2b)(3a+2b) = 3a^2+6ab+2ab+4b^2+a^2 = 4a^2+8ab+4b^2 = 4(a^2+2ab+b^2) = 4\underbrace{(a+b)^2}_{1} = \boxed{4}$

۴

$\log_{(x-1)^2}(x^2-2x+1)+3\log(1-x)=0$ ، $1-x=y$ ، $\log_3^{(-x)}=?$

$\log(-y)^2+3\log y=0 \Rightarrow 2\log y+3\log y=0 \Rightarrow 5\log y=0 \Rightarrow \log y=1 \Rightarrow y=10$

$1-x=10 \Rightarrow x=-9$

$\log_3^{(-9)} = \boxed{2}$

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$\log_2^{(x^2+2x+3)}+\log_2^{(x-2)}=\underbrace{3}_{\log_2^8}$ ، $\log_{\sqrt[3]{2}}^x=?$

$(x^2+2x+3)(x-2)=8 \Rightarrow x^3-8=8 \Rightarrow x^3=16 \Rightarrow x=\sqrt[3]{16}$

$\log_{\sqrt[3]{2}}^x = \log_{\sqrt[3]{2}}^{\sqrt[3]{16}} = \log_{2^{\frac{1}{3}}}^{2^{\frac{4}{3}}} = \frac{\frac{4}{3}}{\frac{1}{3}} = \boxed{4}$

۶

$\log(2-x) - \log\frac{1}{(x-2)^2} = 3 \qquad \log_{\sqrt{2}}^{(-x)}$ $\qquad \to \frac{y}{\frac{1}{y^2}} = y^3$

$2-x=y$

$\log y - \log\frac{1}{(-y)^2} = 3 \Rightarrow \log y - \underbrace{\log\frac{1}{y^2}}_{\log y^{-2}} = 3 \Rightarrow \log\frac{y}{y^{-2}} = 3 \Rightarrow \log y^3 = 3 \Rightarrow y^3 = 10^3 \Rightarrow y = 10$

$3\log y = 3 \quad y = 10$

$y = 2 - x = 10 \Rightarrow x = -8 \qquad \log_{\sqrt{2}}^{(-x)} = \log_{\sqrt{2}}^{+8} = \log_{2^{\frac{1}{2}}}^{2^3} = \frac{3}{\frac{1}{2}} = 6$

۷

$3^{x^2-2} = \underbrace{81^x}_{3^{4x}} \qquad \log_9^{(x-2)}$

$x^2 - 2 = 4x \Rightarrow x^2 - 4x - 2 = 0$

$\Delta = 24 \quad x = \frac{4 \pm \sqrt{24} \to 2\sqrt{6}}{2} = 2 \pm \sqrt{6}$ (غ‌ق‌ق)

$\log_9^{(x-2)} = \log_9^{(2+\sqrt{6}-2)} = \log_9^{6^{\frac{1}{2}}} = \frac{1}{2}$

۸

$\log_3^2 = \frac{5}{8} \qquad \log_{18}^8 = ?$

$\frac{\log_3^8}{\log_3^{18 \to 2\times 3^2}} = \frac{3\log_3^2}{\log_3^2 + \log_3^{3^2}} = \frac{3(\frac{5}{8})}{\frac{5}{8}+2} = \frac{\frac{15}{8}}{\frac{21}{8}} = \frac{15}{21} = \frac{5}{7}$

۹

$\log_4^3 = 0/8 \qquad \log_{12}^{6} \quad {}^{2\times 3}_{3\times 4}$

$\frac{\log_4^6}{\log_4^{12}} = \frac{\log_4^3 + \log_4^2}{\log_4^3 + \log_4^4} = \frac{0/8 + 0/5}{0/8 + 1} = \frac{1/3}{1/8} \simeq 0/72$

$2^{\frac{1}{2\log 2}} = 2^{\frac{1}{2}\log_2^{10}} = 2^{\log_2^{\sqrt{10}}} = \sqrt{10} \qquad \log_2^{2} = 1 \quad = \sqrt{10}$

$\Rightarrow \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$

۱۰

اول ۱۰ را به توان $\frac{1}{2}$ می‌رسانیم تا برابر $2^{\log_2^{\sqrt{10}}}$ می‌شود سپس بین ۲ و $\sqrt{10}$ جنج می‌زنیم و تقارطیلی می‌کنیم $(a\log 2)x^2 + ax + (b\log 2) = 0$

$x=-1 \Rightarrow a\log 2 - a + b\log 2 = 0 \Rightarrow (a+b)(\log 2) - a = 0 \Rightarrow (a+b)(\log 2) = a \Rightarrow a + b = \frac{a}{\log 2}$

$\Rightarrow b = \frac{a}{\log 2} - a \Rightarrow b = a(\frac{1}{\log 2} - 1) \qquad \frac{b}{a} = \frac{1}{\log 2} - 1 = \frac{1 - \log 2}{\log 2}$

$2^{\frac{1}{2}\log_2^{10}} = 2^{\sqrt{10}} \qquad 2^{\log_2^{\sqrt{10}}} = \sqrt{10}\times 1$

$(\sqrt{2})^{\frac{b}{a}} = (2^{\frac{1}{2}})^{\frac{b}{a}} = 2^{\frac{b}{2a}} = 2^{\frac{1}{2}(\frac{1-\log 2}{\log 2})} = 2^{\frac{1-\log 2}{2\log 2}} = 2^{\frac{1}{2\log 2} - \frac{1}{2}} = \frac{2^{\frac{1}{2\log 2}}}{2^{\frac{1}{2}} \sqrt{2}} = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$

$\frac{1-\log 2}{2\log 2} = \frac{1}{2\log 2} - \frac{\log 2}{2\log 2} = \frac{1}{2\log 2} - \frac{1}{2}$