نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۶ کلاس .....

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$f(x) = 2^{Ax+B}$, $g(x) = x^2$, $f(x) = 2^{x-1}$

$f(1) = g(1) \Rightarrow 2^{A+B} = 1 = 2^0 \Rightarrow A+B = 0$

$f(2) = g(2) \Rightarrow 2^{2A+B} = 2^2 \Rightarrow 2A+B = 2$ $\Rightarrow A = 1, B = -1$

$(1, -1)$

$f(x) = 2^{x-1} \Rightarrow f(0) = 2^{-1} = \frac{1}{2}$

$f(x) = 2^{x-1} \Rightarrow f(0) = 2^{-1} = \boxed{\frac{1}{2}}$

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$\log_2^{(4^x+5)} = x+2 \Rightarrow 4^x+15 = 2^{x+2} \Rightarrow (2^2)^x+15 = 2^x \times 2^2 \Rightarrow (2^x)^2+15 = 4 \times 2^x$

$2^x = t \Rightarrow t^2+15 = 4t \Rightarrow t^2-4t+15 = 0 \Rightarrow (t-3)(t-5) = 0 \begin{cases} t = 3 \\ t = 5 \end{cases}$

$\begin{cases} t = 3 \Rightarrow 2^x = 3 \Rightarrow \log_2^{2^x} = \log_2^3 \Rightarrow x = \log_2^3 \\ t = 5 \Rightarrow 2^x = 5 \Rightarrow \log_2^{2^x} = \log_2^5 \Rightarrow x = \log_2^5 \end{cases}$

مجموع جواب‌ها $= \log_2^3 + \log_2^5 = \boxed{\log_2^{15}}$

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$(\log_{21}^3)^2 + \log_{21}^{(147)} \log_{21}^{(343)}$

$\log_{21}^{147} = \log_{21}^{\left(\frac{21}{3}\right)^2} = \log_{21}^{(21)^2} - \log_{21}^3 = 2 - \log_{21}^3$

$\log_{21}^{343} = \log_{21}^{(21)^2} \times 3 = \log_{21}^{21^2} + \log_{21}^3 = 2 + \log_{21}^3$

$(\log_{21}^3)^2 + (2 - \log_{21}^3)(2 + \log_{21}^3) = (\log_{21}^3)^2 + (4 - (\log_{21}^3)^2) = 4$

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$\log_{10}^{x^2-2x+1} + 2\log_{10}^{1-x} = 5 \Rightarrow \log^{(x-1)^2} + 10\log^{(1-x)^2} \Rightarrow$

$\log_{10}^{(x-1)^2(1-x)^2} = 5 \Rightarrow (x-1)^2(1-x)^2 = 10^5 \Rightarrow \underbrace{-(x-1)^5}_{(1-x)^5} = 10^5 \Rightarrow -(x-1)^5 = 10^5 \Rightarrow -(x-1) = 10$

$\Rightarrow -x+1 = 10$

$\Rightarrow x = -9$

$\log_3^{-x} = \log_3^{-(-9)} = 2$

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$\log_2^{(x^2+2x+4)} + \log_2^{(x-2)} = 3 \Rightarrow \log_2^{(x-2)(x^2+x+3)} = 3$

$\Rightarrow \log_2^{x^3-8} = 3 \Rightarrow x^3-8 = 2^3 \Rightarrow x^3 = 16 \Rightarrow x = \sqrt[3]{16}$

$\log_{\sqrt{2}}^{x} \Rightarrow \log_{\sqrt[3]{2}}^{\sqrt[3]{16}} = \log_2^{16} = 4$

۶

$\log(x-3) - \log \frac{1}{(x-2)^2} = 2 \Rightarrow \log_{10}^{-(x-2)} \div \frac{1}{(x-2)^2} = 2 \Rightarrow$

$-(x-2)^2 = 10^2 \Rightarrow -x+2 = 10 \Rightarrow x = -8$

$\log_{\sqrt{2}}^{-(x)} \xrightarrow[\text{مبنا}]{\text{تغییر}} \log_{\sqrt{2}}^{8} \Rightarrow \log_{2^{\frac{1}{2}}}^{2^3} \xrightarrow{\frac{3}{\frac{1}{2}}} \log_2^2 \Rightarrow y = \log_{\sqrt{2}}^{-(x)}$

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$3^{x^2-2} = \underbrace{81}_{3^4}^{x} \Rightarrow x^2-2 = 4x \Rightarrow x^2-4x-2=0 \Rightarrow \Delta = 16-4(-2) = 24$

$x_1 = \frac{4-\sqrt{24}}{2} = \frac{4-2\sqrt{6}}{2} = 2-\sqrt{6}$ غ ق ق

$x_2 = \frac{4+\sqrt{24}}{2} = 2+\sqrt{6}$ ق ق

$\log_{\sqrt{6}}^{(x-2)} = \log_{6}^{\sqrt{6}} = \frac{1}{2}\log_6^6 = \frac{1}{2}$

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$\log_{18}^{8} = \log_{2^{\frac{3}{2}}}^{2^3} = 3$

$\log_2^{18} = \log_2^2 + \log_2^9 = 1 + 2\log_2^3$ ($\log_2^3 = \frac{5}{2}$)

$\log_{18}^{8} = \frac{\log_2^8}{\log_3^{18}} = \frac{3}{1+2\left(\frac{5}{2}\right)} = \frac{3}{6} = \frac{3\times 2}{6} = \frac{5}{7}$

$\log_2^{32} = \frac{5}{8}$

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$\log_{12}^{4} = \frac{\log_2^4}{\log_2^{12}}$ $\log_2^4 = \log_2^2 + \log_2^3 = \log_2^2 + \log_2^{\frac{3}{2}} = \frac{1}{2} + 0.11 = \frac{12}{10}$

$\log_2^{12} = \log_2^2 + \log_2^3 = 1 + 0.18 = \frac{18}{10} \Rightarrow \log_{12}^{4} = \frac{\frac{12}{10}}{\frac{18}{10}} = \frac{12}{18}$

۱۰

$a\log 2 + b\log 2 = a \Rightarrow b\log 2 = a - a\log 2$ تمرین: نشان دهید $\frac{b}{a}$ گویا است.

$\Rightarrow b\log 2 = a(1-\log 2) \Rightarrow \frac{b}{a} = \frac{1-\log 2}{\log 2} = \frac{\log 10 - \log 2}{\log 2} = \frac{\log 5}{\log 2}$

$\frac{b}{a} = \log_2^5$

$\sqrt{2}^{\frac{b}{a}} = \left(\sqrt{2}\right)^{\log_2^5} = 5^{\log_2^{\sqrt{2}}} = 5^{\frac{1}{2}\log_2^2} = 5^{\frac{1}{2}} = \sqrt{5}$