زهرا یزدانیان

به نام خدا

۱) $f(x) = 3^{Ax+B}$

$y = x^2 \longrightarrow x=1 \longrightarrow y=1$

$\searrow x=3 \longrightarrow y=9$

$3^{Ax+B} \longrightarrow 3^{3A+B} = 3^2 \longrightarrow 3A+B=2$

$3^{Ax+B} \longrightarrow 3^{A+B} = 3^0 \longrightarrow A+B=0$

$\left.\begin{array}{l} 2A=2 \rightarrow A=1 \\ B=-1 \end{array}\right.$

$x=0 \longrightarrow f(x) = 3^{0+B} = 3^{-1} = \boxed{\frac{1}{3}}$

---

۲) $\log_2(4^x+15) = x+3 \longrightarrow 2^{x+3} = 4^x+15 \longrightarrow 8\cdot 2^x = 2^{2x}+15 \longrightarrow 8\cdot 2^x - 2^{2x} - 15 = 0$

$\downarrow (2^x)$

$t^2 - 8t + 15 = 0 \longrightarrow (t-3)(t-5) = 0 \longrightarrow \begin{cases} t=3 \rightarrow 2^x = 3 \rightarrow x = \log_2^3 \\ t=5 \longrightarrow 2^x = 5 \rightarrow x = \log_2^5 \end{cases}$

$\log_2^5 + \log_2^3 = \boxed{\log_2^{15}}$

---

۳) $(\log_{21}^3)^2 + \log_{21}^{147} \times \log_{21}^{343} = (\log_{21}^3)^2 + \log_{21}^{147} \times (\underbrace{\log_{21}^7 + \log_{21}^3}_{2} + \log_{21}^3)$

$= (\log_{21}^3)^2 + \log_{21}^{147} \times 2 + \log_{21}^{147} \times \log_{21}^3 = (\log_{21}^3)^2 + 2 + 2\log_{21}^7 + \log_{21}^3 + \log_{21}^3 \times \log_{21}^7$

$\log_{21}^{147} \downarrow \log_{21}^{21} + \log_{21}^7$

$(\log_{21}^3)^2 \rightarrow \log_{21}^3 \times \log_{21}^3$

$2\log_{21}^7 + \log_{21}^3 \rightarrow \log_{21}^7 + 1$

$\log_{21}^3(\underbrace{\log_{21}^3 + \log_{21}^7}_{1}) + 2 + \log_{21}^7 + 1 = 3 + \underbrace{\log_{21}^3 + \log_{21}^7}_{1} = \boxed{4}$

---

۴) $\log(x-1)^2 + 2\log(11-x) = 5 \longrightarrow \log \frac{(1-x)^2 \times (11-x)^2}{} = \log 10^5$

$(1-x)^5 = 10^5 \longrightarrow 1-x = 10 \longrightarrow x = -9 \longrightarrow \log_2^{(-9)^2} = \boxed{4}$

---

۵) $\log_2^{(x^2-2x+1)(x-2)} = 3 \longrightarrow x^3 + 2x^2 - x^2 - 2x + x - 1 = 8 \longrightarrow x^3 = 10$

$\log_{\sqrt[3]{10}}^{\sqrt[3]{10}} = \log_{\frac{1}{2^3}}^{2^3} = \frac{3}{-3}\log_2^2 = \boxed{-1}$

---

۶) $\log(2-x) - \log\frac{1}{(2-x)^2} = 3 \longrightarrow -(x-2)^3 = 10^3 \longrightarrow 2-x = 10 \longrightarrow x = -8 \longrightarrow \log_{\sqrt[3]{8}}^{8} = \boxed{6}$

۷) $\log_4^{(x-2)} = \log_4^{2+\sqrt{4}-2} = \log_4^{\sqrt{4}} = \boxed{\frac{1}{2}}$

$3^{x^2-2} = 3^{4x} \longrightarrow x^2-2=4x \longrightarrow x^2-4x-2=0 \longrightarrow \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{4 \pm 2\sqrt{6}}{2} \longrightarrow 2 \pm \sqrt{6}$

فقط + قابل قبول است

---

۸) $\log_3^2 = \frac{a}{8} \longrightarrow \log_{18}^8 = \frac{\log_3^8}{\log_3^{18}} = \frac{3\log_3^2}{2\log_3^3 + \log_3^2} = \frac{\frac{15}{8}}{\frac{21}{8}} = \frac{15}{21} = \boxed{\frac{5}{7}}$

---

۹) $\log_{12}^6 = \frac{\log_6^6}{\log_6^{12}} = \frac{\log_6^2 + \log_6^3}{\log_6^6 + \log_6^3} = \frac{0/778 + 0/613}{1 + 0/778} = \boxed{\frac{13}{18}}$

---

۱۰) $a\log 2 + b\log 2 = a \longrightarrow \log 2(a+b) = a \longrightarrow \log 2 = \frac{a}{a+b} \longrightarrow \log_2^{10} = \frac{a+b}{a} = 1 + \frac{b}{a}$

$\frac{b}{a} = \log_2^{10} - 1 = \log_2^5 \longrightarrow \sqrt{2}^{\log_2^5} = 5^{\log_2^{\sqrt{2}}} = 5^{\frac{1}{2}} = \boxed{\sqrt{5}}$