①

$3^{Ax+B} = x^2 \xrightarrow{x=1} 3^{A+B}=1 \to A+B=0 \to B=-A$ ★

$3^{Ax+B} = x^2 \xrightarrow{x=3} 3^{3A+B}=9 \to 3A+B=2 \xrightarrow{★} 2A=2 \to A=1$

$B=-1$

$\Rightarrow 3^{Ax+B} = 3^{x-1} \xrightarrow{x=0} 3^{-1}=\boxed{\frac{1}{3}} \to$ عرض محل تلاقی تابع با محور ها

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②

$\log_2(4^x+15) = x+3 \to 4^x+15=2^{(x+3)} \to 2^{2x}+15=2^x\times 8$

$\Rightarrow 2^x\times 8-2^{2x}=15 \to 2^x(8-2^x)=15 \xrightarrow{2^x=t} -t^2+8t=15$

$\Rightarrow t^2-8t+15=0 \to (t-3)(t-5)=0 \to t=3,5 \to 2^x=3 , 2^x=5$

$\to x=\log_2 3$ و $x=\log_2 5$ $\xrightarrow{\text{جمع جواب}} \log_2 3+\log_2 5=\boxed{\log_2 15}$

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③

$(\log_{21} 3)^2+\log_{21}(21\times 7)\times\log_{21}(21\times 21\times 3)=$ [illegible]

$\Rightarrow \log_{21}(21\times 7)\times\log_{21}(21\times 21\times 3) \to (1+\log_{21}7)(2+\log_{21}3)$

$= 2+\underbrace{\log_{21}3+\log_{21}7}_{\log_{21}21=1}+\log_{21}7+(\log_{21}7\times\log_{21}3)$

$\Rightarrow 3+\underbrace{\log_{21}7}_{\underbrace{\log_{21}21-\log_{21}3}_{1-t}}(1+\underbrace{\log_{21}3}_{t}) \xrightarrow{\log_{21}3=t} t^2+3+(1-t)(1+t)$

$\Rightarrow t^2+3+1-t^2=\boxed{4}$

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④

$x^2-2x+1=(x-1)^2=(1-x)^2 \to \log(1-x)^2+\log(1-x)^3=5$

$\Rightarrow \log_{10}(1-x)^5=5 \to 10^5=(1-x)^5 \to 1-x=10 \to \boxed{x=-9}$

$\log_3 9=\boxed{2}$

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$\log_{2}^{(x-2)(x^2+2x+4)} = 3 \to \log_2^{x^3-8} = 3 \to x^3 - 8 = 8$

$\Rightarrow x^3 = 16 \to x = 2^{\frac{4}{3}} \to \log_{2^{\frac{1}{3}}}^{2^{\frac{4}{3}}} = \frac{4}{3} \times 3 \times \log_2^2 = \boxed{4}$

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$(2-x)^2 = (x-2)^2$ ★

$\log(2-x) - \log_{0/1}(2-x)^2 = 3 \to \log(2-x) + \log(2-x)^2 = 3$

$\Rightarrow \log_{10}(2-x)^3 = 3 \to (2-x) = 10 \to 2 - x = 10 \to x = -8$

$\log_{\sqrt{2}}^{8} = \log_{2^{\frac{1}{2}}}^{2^3} = 6 \log_2^2 = \boxed{6}$

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$3^{x^2-2} = 81^x \to 3^{x^2-2} = 3^{4x} \to x^2 - 2 = 4x$

$\Rightarrow x^2 - 4x - 2 = 0 \qquad \Delta = 16 + 8 = 24 \qquad x = \frac{4 \pm 2\sqrt{6}}{2}$

$x \to 2 - \sqrt{6}$ غ ق ق

$x \to 2 + \sqrt{6}$ ✓

$x^2 - 2 > 0 \to x^2 > 2 \to x > \sqrt{2}, \ x < -\sqrt{2}$

$(2-\sqrt{6})^2 = 4 + 6 - 4\sqrt{6} > 2 \to 8 - 4\sqrt{6} > 0$ ✗

$4\sqrt{6} > 8$

$\log_6^{\sqrt{6}} = \boxed{\frac{1}{2}}$

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$\log_{18}^{8} = \frac{\log_3^8}{\log_3^{18}} = \frac{3\log_3^2}{\log_3^9 + \log_3^2} = \frac{\frac{15}{8}}{\frac{21}{8}} = \boxed{\frac{5}{7}}$

$2 + \frac{5}{8} = \frac{21}{8}$

۹

$\log_4^3 = 0/8 \to \frac{1}{2}\log_2^3 = 0/8 \to \log_2^3 = 1/6$

$\log_{12}^{6} = \frac{\log_2^6}{\log_2^{12}} = \frac{\log_2^3 + \log_2^2}{\log_2^4 + \log_2^3} = \frac{2/6}{3/6} = \frac{\frac{26}{10}}{\frac{36}{10}} = \frac{13}{18}$

$x=-1 \rightarrow a+c=b \rightarrow a\log ۳ + b\log ۳ = a$

$\Rightarrow (a+b)\log ۳ = a \rightarrow \log ۳ = \frac{a}{a+b} \Rightarrow \frac{1}{\log^{۳}_{۱۵}} = \frac{a+b}{a}$

$\Rightarrow \log^{۱۵}_{۳} = 1+\frac{b}{a} \rightarrow \frac{b}{a} = \log^{۱۵}_{۳} - \log^{۳}_{۳} = \log^{۵}_{۳}$

$(\sqrt{۳})^{\frac{b}{a}} = (۳^{\frac{1}{۲}})^{\log^{۵}_{۳}} = ۳^{\log^{\sqrt{۵}}_{۳}} = \sqrt{۵}^{\log^{۳}_{۳}} = \sqrt{۵}$