①

$y = x^2 \xrightarrow{x=1} y=1$

$\xrightarrow{x=3} y=9$

$f(x) = 3^{Ax} + B \quad (1,1) \rightarrow 1 = 3^{A+B} \rightarrow A + B = 0$

$(2,9) \rightarrow 9 = 3^{2A+B} \rightarrow 2A + B = 2$

$\boxed{A = 1}$ $\boxed{B = -1}$

$f^{-1}(x) = 3^{x-1} \xrightarrow{x=0} \boxed{\frac{1}{3}}$

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②

$\log_2^{(4^x+15)} = x + 3 \rightarrow 2^{x+3} = 4^x + 15$

$\rightarrow 2^x \times 8 = (2^x)^2 + 15 \quad (t)$

$t^2 - 8t + 15 \rightarrow (t-3)(t-5)$

$t = 5 \quad t = 3$

$2^x = 5 \quad 2^x = 3$

$x_1 = \log_2^5 \quad x_2 = \log_2^3$

$x_1 + x_2 = \log_2^5 + \log_2^3 \Rightarrow \boxed{\log_2^{15}}$

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③

$(\log_{21}^3)^2 + \log_{21}^{(147)} \log_{21}^{(1323)} \qquad \log_{21}^7 = 1 - \log_{21}^3$

$A = (\log_{21}^3)^2 + (\log_{21}^{21} + \log_{21}^7)(\log_{21}^{21^2} + \log_{21}^3) \rightarrow A = (\log_{21}^3)^2 + (1 + \log_{21}^7)(2\log_{21}^{21} + \log_{21}^3)$

$(\log_{21}^3)^2 + (1 + 1 - \log_{21}^3)(2 + \log_{21}^3) = (4 - (\log_{21}^3)^2) + (\log_{21}^3)^2 = 4$

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④

$5^{(x^2-2x+1)} + 4\cdot 5^{(1-x)} = 5 \rightarrow 5^{(1-x)^2} + 4\cdot 5^{(1-x)} - 5 = 0 \rightarrow \log_{10}^{(1-x)} = -1$

$0/1 = 1 - x \rightarrow \boxed{x = 0/9}$

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⑤

$\log_2^{(x^2+2x+4)} + \log_2^{(x-2)} = 3$

$(x^2 + 2x + 4)(x - 2) = 8$

$x^3 - 8 = 8 \rightarrow x^3 = 16 \qquad \log_{2^{1/3}}^x = 3\log_2^x =$

$\log_2^{x^3} = \log_2^{16} = 4$

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$\log^{(2-x)} - \log^{\frac{1}{(x-2)^2}} = 3 \rightarrow \log^{(4-x)} - \log^{(2-x)^{-2}} = 3 \rightarrow \log^{2-x} + 2\log^{2-x} = 3$

$3\log^{2-x} = 3 \rightarrow x = -8$

$\log_{24}^{8} = (16)$

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if: $3^{x^2-2} = 81^x \rightarrow 3^{x^2-2} = 3^{4x} \rightarrow x^2 - 4x = 2$

$(x-2)^2 - 4 = 2 \rightarrow (x-2)^2 = 6 \rightarrow x - 2 = \pm\sqrt{6}$ $\begin{cases} x = 2-\sqrt{6} < 2 \ \times \\ x = 2+\sqrt{6} \end{cases}$

$\log_6^{x-2} \xrightarrow{x=\sqrt{6}+2} \log_6^{2+\sqrt{6}-2} = \frac{1}{2}$

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$\log_3^2 = \frac{5}{8}$ $\quad \log_{18}^{8} = ? \rightarrow \frac{\log_3^8}{\log_3^{18}} \rightarrow \frac{3\log_3^2}{2\log_3^3 + \log_3^2} \rightarrow \frac{\frac{15}{8}}{\frac{16+5}{8}} \rightarrow \frac{15}{21} \Rightarrow \frac{5}{7}$

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$\log_4^3 = 0.8 \rightarrow \frac{1}{2}\log_2^3 = 0.8 \rightarrow \log_2^3 = 1.6$

$\log_{12}^{6} \rightarrow \frac{\log_2^6}{\log_2^{12}} = \frac{\log_2^2 + \log_2^3}{2\log_2^2 + \log_2^3} \rightarrow \frac{1+1.6}{2+1.6} \rightarrow \frac{2.6}{3.6} \rightarrow \frac{1.3}{1.8}$

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$(a\log^2)x^2 + ax + b\log^2 = 0$

شادی محقق

$\log^2(a+b) = a \qquad \frac{a+b}{a} \times \log 2 = 1 \rightarrow (1+\frac{b}{a})\log 2 = 1$

$\log_2^{(1+\frac{b}{a})} = 1 \rightarrow 10 = 2 \times 2^{\frac{b}{a}} \rightarrow 2^{\frac{b}{a}} = 5 \rightarrow (\sqrt{2})^{\frac{b}{a}} =$

$2^{(\frac{1}{2})(\frac{b}{a})} = (2^{\frac{b}{a}})^{\frac{1}{2}} \rightarrow 5^{\frac{1}{2}} \Rightarrow \sqrt{5}$