بردیا نوروزی نسب یازدهم B

۲۰

سؤال ۱)

$f(x) = 3^{Ax+B}$ $\quad y = x^2$

$f(1) = y(1) \Rightarrow 3^{A+B} = 1 \Rightarrow \overbrace{A+B=0}$ (رابطه ۱

$f(3) = y(3) \Rightarrow 3^{3A+B} = 9 \Rightarrow \overbrace{3A+B=2}$ (رابطه ۲

$$\begin{cases} -A-B=0 \\ 3A+B=2 \\ \hline 2A=2 \end{cases} \Rightarrow \begin{cases} B=-1 \\ A=1 \end{cases}$$

۵

$\xrightarrow{\text{جایگذاری}} f(x) = 3^{x-1} \xrightarrow{f(0)} f(0) = 3^{-1} \Rightarrow \boxed{\frac{1}{3}}$ جواب

سؤال ۲)

$\log_2^{(4^x+15)} = x+3 \Rightarrow 2^{x+3} = 4^x + 15 \Rightarrow \overset{t}{(2^x)} \times 8 = (\overset{t}{2^x})^2 + 15$

۵

$\Rightarrow t^2 - 8t + 15 \Rightarrow (t-3)(t-5)$ $\begin{cases} t_1 = 3 \\ t_2 = 5 \end{cases}$ $\begin{cases} 2^x = 3 \Rightarrow x_1 = \log_2^3 \\ 2^x = 5 \Rightarrow x_2 = \log_2^5 \end{cases}$

$x_1 + x_2 = \log_2^3 + \log_2^5 = \log_2^{15}$

سؤال ۳)

$(\log_{21}^3)^2 + \log_{21}^{(147)} \log_{21}^{(1323)} = (\log_{21}^3)^2 + \log_{21}(21\times7)(\log_{21}^{21^2\times3})$

$\Rightarrow (\log_{21}^3)^2 + (\cancel{\log_{21}^{21}}^{1} + \log_{21}^7)(\underbrace{\log_{21}^{21^2}}_{2\log_{21}^{21}=2} + \log_{21}^3) = (\log_{21}^3)^2 + (1+\log_{21}^7)\underbrace{(2\times1+\log_{21}^3)}_{(X)}$

۵

$\Rightarrow (\log_{21}^3)^2 + 2 + \log_{21}^3 + 2\log_{21}^7 + \log_{21}^7 \times \log_{21}^3$

$\xrightarrow{\text{فاکتور}} \log_{21}^3(\log_{21}^3 + \log_{21}^7) + 2 + \log_{21}^3 + 2\log_{21}^7 = \log_{21}^3 + 2 + \log_{21}^3 + 2\log_{21}^7$

$\Rightarrow 2\log_{21}^3 + 2 + 2\log_{21}^7 = 2(\log_{21}^3 + \log_{21}^7) + 2 = 2\log_{21}^{21} + 2 = 2+2 = 4$ جواب

سوال ۴)

$\log(x^2-2x+1)+3\log(1-x)=5 \qquad \log_3(-x)=?$

۲

$\log(1-x)^2+\log(1-x)^3=5 \Rightarrow \log_{10}-(x-1)^5=5 \Rightarrow (1-x)^5=10^5$

$\Rightarrow 1-x=10 \Rightarrow x=-9 \qquad \log_3(9)=2$

سوال ۵)

$\log_2(x^2+2x+4)+\log_2(x-2)=3 \Rightarrow \log_2(x^2+2x+4)(x-2)=3$

$x^3-8=8 \Rightarrow x^3=16 \Rightarrow x=\sqrt[3]{16}$ / $\log_{\sqrt[3]{2}}\sqrt[3]{16} \Rightarrow \frac{1}{3}\times 3\log_2 16$

$\Rightarrow \log_2 16 = 4$ }

سوال ۶)

$\log(2-x)-\log\frac{1}{(x-2)^2}=3 \Rightarrow \log(2-x)-\log(2-x)^{-2}=3$

$\Rightarrow \begin{cases} 3\log(2-x)=3 \Rightarrow x=-8 \\ \log(2-x)+2\log(2-x)=3 \end{cases}$ $\log_{\sqrt2}(-x)=?$ $\log_{\sqrt2}8=\log_{2^{\frac12}}2^3=6$ }

سوال ۷)

$3^{x^2-2}=81^x \Rightarrow 3^{x^2-2}=3^{4x} \Rightarrow x^2-2=4x \Rightarrow x^2-4x-2=0$

$x^2-4x=2 \Rightarrow x^2-4x+4=6 \Rightarrow (x-2)^2=6 \Rightarrow x-2=\pm\sqrt6$ $\log_6 \overset{>0}{(x-2)}=?$

$\Rightarrow \log_6(x-2)=\log_6\sqrt6=\frac12\log_6 6=\frac12$ }

سوال ۸)

$\log_3 2=\frac58$ $\log_{18}8=? \Rightarrow \frac{\log_3 8}{\log_3 18}=\frac{3\log_3 2}{2\log_3 3+\log_3 2} \Rightarrow \frac{\frac{15}{8}}{\frac{16+5}{8}}=\frac57$ }

سوال ۹)

$\log_4 3=0.8$ $\log_{12}6=?$ $\log_4 3=\frac{1}{\log_3 4}=\frac{1}{\log_3 2^2}=\frac{1}{2\log_3 2}=\frac{8}{10}$

$\Rightarrow \frac{1}{\log_3 2}=\frac85$ / $\log_{12}6=\frac{1}{\log_6 12}=\frac{1}{\log_6 6+\log_6 2}=\frac{1}{1+\frac{1}{\log_2 6}}=\frac{1}{1+\frac{1}{\log_2 2+\log_2 3}}$

$\frac{1}{1+\frac{1}{1+\frac85}}=\frac{1}{1+\frac{5}{13}}=\frac{13}{18}$ }

$$(a - \log ٢)x^٢ + ax + b\log_٢ = ٠ \qquad \log^٢(a+b) = a$$

(۱۰ گ

$$\log^٢\left(1+\frac{b}{a}\right) = 1 \Rightarrow ۱۰ = ٢ \times ٢^{b/a} \Rightarrow ٢^{b/a} = ۵ \Rightarrow (\sqrt{٢})^{b/a} = ٢^{(۱/٢)(b/a)}$$

$$\frac{a+b}{a} \times \log ٢ = 1 \rightarrow \left(1+\frac{b}{a}\right)\log ٢ = 1$$

$$\Rightarrow \left(٢^{b/a}\right)^{۱/٢} \Rightarrow ۵^{۱/٢} = \sqrt{۵}$$