ریاضی تجربی یازدهم B

سؤال ۱)

$f(x) = 3^{Ax+B} \qquad y = x^2$

$f(1) = y(1) \Rightarrow 3^{A+B} = 1 \Rightarrow A+B=0$ رابطه ۱)

$f(3) = y(3) \Rightarrow 3^{3A+B} = 9 \Rightarrow 3A+B=2$ رابطه ۲)

$$\begin{cases} -A-B=0 \\ 3A+B=2 \end{cases} \quad 2A=2 \Rightarrow \begin{cases} B=-1 \\ A=1 \end{cases}$$

جایگذاری $\rightarrow f(x) = 3^{x-1} \xrightarrow{f(0)} f(0) = 3^{-1} \Rightarrow \boxed{\frac{1}{3}}$ جواب

سؤال ۲)

$\log_2 (4^x+15) = x+3 \Rightarrow 2^{x+3} = 4^x + 15 \Rightarrow \overset{t}{(2^x)} \times 8 = (\overset{t}{2^x})^2 + 15$

$\Rightarrow t^2 - 8t + 15 \Rightarrow (t-3)(t-5) \begin{cases} t_1 = 3 \\ t_2 = 5 \end{cases} \begin{cases} 2^x = 3 \Rightarrow x_1 = \log_2 3 \\ 2^x = 5 \Rightarrow x_2 = \log_2 5 \end{cases}$

$x_1 + x_2 = \log_2 3 + \log_2 5 = \log_2 15$

سؤال ۳)

$(\log_{21} 3)^2 + \log_{21}(147) \log_{21}(1323) = (\log_{21} 3)^2 + \log_{21}(21 \times 7)(\log_{21} 21^2 \times 3)$

$\Rightarrow (\log_{21} 3)^2 + (\cancel{\log_{21} 21}^{\,1} + \log_{21} 7)(\underbrace{\log_{21} 21^2}_{2\log_{21} 21 = 2} + \log_{21} 3) = (\log_{21} 3)^2 + \underbrace{(1+\log_{21} 7)(2\times 1 + \log_{21} 3)}_{(\times)}$

$\Rightarrow (\log_{21} 3)^2 + 2 + \log_{21} 3 + 2\log_{21} 7 + \log_{21} 7 \times \log_{21} 3$

فاکتور $\Rightarrow \log_{21} 3 (\log_{21} 3 + \log_{21} 7) + 2 + \log_{21} 3 + 2\log_{21} 7 = \log_{21} 3 + 2 + \log_{21} 3 + 2\log_{21} 7$

$\Rightarrow 2\log_{21} 3 + 2 + 2\log_{21} 7 = 2(\log_{21} 3 + \log_{21} 7) + 2 = 2\log_{21} 21 + 2 = 2+2 = 4$

جواب

سوال ۴)

$$\log(x^{۲}-۲x+۱)+۳\log(۱-x)=۵ \qquad \log_{۳}(-x)=?$$

$$\log(۱-x)^{۲}+\log(۱-x)^{۳}=۵ \Rightarrow \log_{۱۰}-(x-۱)^{۵}=۵ \Rightarrow (۱-x)^{۵}=۱۰^{۵}$$

$$\Rightarrow ۱-x=۱۰ \Rightarrow x=-۹ \qquad \log_{۳}(۹)=۲$$

سوال ۵)

$\log_2 (x^2+2x+4) + \log_2 (x-2) = 3 \Rightarrow \log_2 (x^2+2x+4)(x-2) = 3$

$x^3 - 8 = 8 \Rightarrow x^3 = 16 \Rightarrow x = \sqrt[3]{16} \;/\; \log_{\sqrt[3]{2}} \sqrt[3]{16} \Rightarrow \frac{1}{3} \times 3 \log_2 16$

$\Rightarrow \log_2 16 = 4$

سوال ۶)

$\log(2-x) - \log \frac{1}{(x-2)^2} = 3 \Rightarrow \log(2-x) - \log (2-x)^{-2} = 3$

$\Rightarrow \begin{cases} 3\log(2-x) = 3 \Rightarrow x = -8 \\ \log(2-x) + 2\log(2-x) = 3 \end{cases}$ $\quad \log_{\sqrt{2}}(-x) = ? \quad \log_{\sqrt{2}} 8 = \log_{2^{\frac{1}{2}}} 2^3 = 6$

سوال ۷)

$3^{x^2-2} = 81^x \Rightarrow 3^{x^2-2} = 3^{4x} \Rightarrow x^2 - 2 = 4x \Rightarrow x^2 - 4x - 2 = 0$

$x^2 - 4x = 2 \Rightarrow x^2 - 4x + 4 = 6 \Rightarrow (x-2)^2 = 6 \Rightarrow x - 2 = \pm\sqrt{6} \quad \log_6 (x-2) = ?$

$\Rightarrow \log_6 (x-2) = \log_6 \sqrt{6} = \frac{1}{2} \log_6 6 = \frac{1}{2}$

سوال ۸)

$\log_3 2 = \frac{5}{8} \quad \log_{18} 8 = ? \Rightarrow \frac{\log_3 8}{\log_3 18} = \frac{3\log_3 2}{2\log_3 3 + \log_3 2} \Rightarrow \frac{\frac{15}{8}}{\frac{16+5}{8}} = \frac{5}{7}$

سوال ۹)

$\log_4 3 = 0/8 \quad \log_{12} 6 = ? \quad \log_4 3 = \frac{1}{\log_3 4} = \frac{1}{\log_3 2^2} = \frac{1}{2\log_3 2} = \frac{8}{10}$

$\Rightarrow \frac{1}{\log_3 2} = \frac{8}{5} \;/\; \log_{12} 6 = \frac{1}{\log_6 12} = \frac{1}{\cancel{\log_6 6} + \log_6 2} = \frac{1}{1 + \frac{1}{\log_2 6}} = \frac{1}{1 + \frac{1}{\log_2 2 + \log_2 3}}$

$\frac{1}{1 + \frac{1}{1 + \frac{8}{5}}} = \frac{1}{1 + \frac{5}{13}} = \frac{13}{18}$

حل ۱۰)

$$(a - \log 2)x^2 + ax + b\log_2 = 0 \qquad \log 2\,(a+b) = a$$

$$\log 2\left(1+\frac{b}{a}\right) = 1 \Rightarrow 10 = 2 \times 2^{b/a} \Rightarrow 2^{b/a} = 5 \Rightarrow (\sqrt{2})^{\frac{b}{a}} = 2^{(1/2)(b/a)}$$

$$\frac{a+b}{a} \times \log 2 = 1 \Rightarrow \left(1+\frac{b}{a}\right)\log 2 = 1$$

$$\Rightarrow \left(2^{b/a}\right)^{1/2} \Rightarrow 5^{1/2} = \sqrt{5}\,\}$$