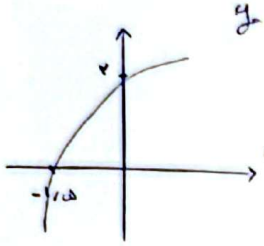
سوال ۱ ⇐



$y = 1 - \log_c^{(ax-b)}$ ، $b+c=-\frac{3}{4}$ ، $(a+c)b=?$

$(0,2) \to 1-\log_c^{-b}=2 \to \log_c^{-b}=-1 \to -b=\frac{1}{c}$ ★

$(-\frac{3}{2},0) \to 1-\log_c^{-\frac{3}{2}a-b}=0 \to -\frac{3}{2}a-b=c \to -\frac{3}{2}a=b+c \Rightarrow a=1$

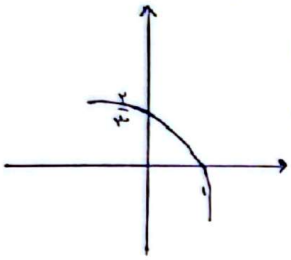
$b+c=-\frac{3}{4} \xrightarrow{\times\frac{1}{c}} -\frac{1}{c}+c=-\frac{3}{4} \to \frac{c^2-1}{c}=-\frac{3}{4} \to 4c^2+3c-4=0$

$c'+c''=-\frac{3}{4}$ ، $c'c''=-1$ $\to$ $c=\frac{1}{2}$ یا $c=-2$

$-b=\frac{1}{c} \xrightarrow{c=\frac{1}{2}} b=-2$

$(a+c)b=(1+\frac{1}{2})\times(-2)=-3$

سوال ۲ ⇐



$f(x)=1+c\times 3^{ax+b}$ ، $f(-1)=?$

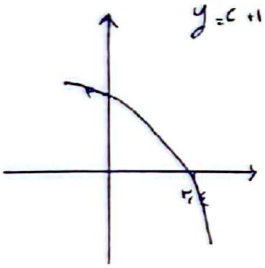
$(1,0) \to 1+c\times 3^{a+b}=0 \to c\times 3^{a+b}=-1$

$(0,\frac{2}{3}) \to 1+c\times 3^{a}=\frac{2}{3} \to c\times 3^{a}=-\frac{1}{3}$ ★

$\div \to 3^b=3 \to b=1$

$f(x)=1+c\times 3^{a}\times 3^{x} \xrightarrow{f(-1)} 1-\frac{1}{3}\times 3^{-1}=1-\frac{1}{9}=\frac{8}{9}$

سوال ۳ ⇐



$y=c+\log_3^{(ax+b)}$ ، $\frac{a}{b}=?$

$(0,2) \to c+\log_3^{b}=2$

$(2,4,0) \to (c+\log_3^{2,4a+b}=0)^{x-1}$

$+ \to \log_3^{b}-\log_3^{2,4a+b}=2$

$\log_3^{\frac{b}{2,4a+b}}=2 \to \frac{b}{2,4a+b}=9$

$b=5a+3b \to 3b=-5a$

$\frac{a}{b}=\frac{-3}{5}=-\frac{3}{5}$

سوال ۴ ⇐

$f(x)=\log_{\frac{1}{3}}^{(|x^2-4|-x)} \to |x^2-4|-x>0$

① $x^2-4\geq 0 \to x^2\geq 4 \to x\leq -2$ یا $x\geq 2$ (I)

$x^2-4<x \to x^2-x-4<0 \to$ ریشه‌ها: $\frac{1\pm\sqrt{17}}{2}$ (II)

$x>\frac{1-\sqrt{17}}{2}$ ∪ $x<2$ ⇒ $-2<x<2$ ∪ $x\geq 2$ ⇒ $x>-2$ ... (۱)

② $x^2-4<0 \to x^2<4 \to -2<x<2$ (III)

$x^2-4\geq x \to x^2-x-4\geq 0 \to (x-1)(x+3)\geq 0 \to x\leq -3$ یا $x\geq 1$

$x\leq \frac{1-\sqrt{17}}{2}$ ∪ $x\geq 2$ ⇒ $x\leq \frac{1-\sqrt{17}}{2}$ ... (۲)

① ∪ ② → $D_f=(-\infty,\frac{1-\sqrt{17}}{2}]\cup(2,+\infty)$

سوال ۵ ⇐

$f(x)=2+2^{b-ax}$ ، $g(x)=-x^2-2x+a$ ، $(1,?)$ ، $f^{-1}(10)=1$ ، $2b-a=?$

$2+2^{b-a}=-1-2+8 \to 2^{b-a}=2 \to b-a=1$

$f^{-1}(10)=-1$ ⇐ صادق است $(-1,10)$ نقطه $f(x)$ ، پس ⟵ $2+2^{b+a}=10$

$2^{b+a}=8=2^3 \to b+a=3$

$b-a=1$

$b+a=3$

$2b=4 \to b=2$

$a=1$

$2b-a=2(2)-1=3$

سوال ۴ ⇐

$f(x) = -2 + (\frac{1}{2})^{Ax+b}$ ⟶ $(1,?)$, $(2,?)$ ، $f(3) = ?$

$y = x^2 - x$

$x^2 - x \xrightarrow{x=1} 1-1=0 \to (1,0)$; $\xrightarrow{x=2} 4-2=2 \to (2,2)$ ⟶ این نقاط در $f(x)$ صدق می‌کنند

$(1,0) \to -2 + (\frac{1}{2})^{A+B} = 0 \to (\frac{1}{2})^{A+B} = 2 \to (A+B = -1) \times -1$

$(2,2) \to -2 + (\frac{1}{2})^{2A+B} = 2 \to (\frac{1}{2})^{2A+B} = 4 \to 2A+B = -2$

$\to A = -1$, $B = 0$

$f(x) = -2 + (\frac{1}{2})^{-x}$ ⟶ $f(3) = -2 + (\frac{1}{2})^{-3} = -2 + 8 = 6$

سوال ۷ ⇐

$m = m_0 \times (\frac{8}{9})^t$ ⟶ $\frac{1}{4} m_0 = m_0 (\frac{8}{9})^t \Rightarrow \frac{1}{4} = (\frac{8}{9})^t \to \log \frac{1}{4} = \log (\frac{8}{9})^t$

$\log 1 - \log 4 = t(\log 8 - \log 9)$

$-(\log 2^2) = t(3\log 2 - (2\log 3 + \log 2))$

$-2 = t(3\log 2 - 2\log 3 - \log 2)$

$-2 = t(2\log 2 - 2\log 3)$

$-2 = t \times 2(\log 2 - \log 3)$

$t = \frac{-2}{2(\log 2 - \log 3)} = \frac{-2}{2(-0.48)} = \frac{19}{2}$

دقیقه $t = \frac{19}{2} \times 60 = 570$

$\log_3 8 = \frac{1}{\log_8 3} = \frac{1}{\frac{1}{3}} = 3$

$\log_8 2 = \frac{1}{\log_2 8} = \frac{1}{3}$

سوال ۸ ⇐

$m = m_0 \times (\frac{7}{8})^t$ ⟶ $\frac{1}{8} m_0 = m_0 (\frac{7}{8})^t \to \log_{\frac{7}{8}} \frac{1}{8} = t$

$\log_{\frac{7}{8}} \frac{1}{8} = t(\log_{\frac{7}{8}} 7 - \log_{\frac{7}{8}} 8)$

$-3 = t(-1 - (-3))$

$-3 = t(-1 + 3)$

$-3 = 2t$

$t = -\frac{3}{2}$

روز $t = 1.5 \times 7 = 10.5$

$\log_{\frac{7}{8}} 8 = \log_{\frac{8}{7}} 8 = 3$

$\log_{\frac{7}{8}} 7 = \log_{\frac{8}{7}} 7^{-1} = -1$

سوال ۹ ⇐

$a = a_0 \times (\frac{96}{100})^n$ ⟶ $\frac{1}{2} a_0 = a_0 (\frac{96}{100})^n \to \frac{1}{2} = (\frac{96}{100})^n$

$\log \frac{1}{2} = \log (\frac{96}{100})^n \to \log \frac{1}{2} = n \log \frac{96}{100}$

$-\log 2 = n(\log 96 - \log 100)$

$-0.3 = n(1.98 - 2)$

$n = \frac{-0.3}{-0.02} = 15$

سوال ۱۰ ⇐

$y = \log x^2$ رسم کنید

x	y
-1	0
$-\frac{1}{2}$	-2
1	0
2	2

$x^2 = 9 \log_x 9$ ، $y = \log_x 9$ رسم کنید

$x > 0$

x	y
1	0
$\frac{1}{2}$	-1
3	1