

$$f(x) = 1 - \log_c(ax-b) \Rightarrow f(0) = 2 \Rightarrow 1 - \log_c(-b) = 2 \Rightarrow \log_c(-b) = -1 \Rightarrow -b = \frac{1}{c} \Rightarrow b = -\frac{1}{c}$$

$$f(-\frac{c}{4}) = 0 \Rightarrow 1 - \log_c(-\frac{c}{4}a - b) = 0 \Rightarrow \log_c(-\frac{c}{4}a - b) = 1 \Rightarrow c = -\frac{c}{4}a - b$$

$$\Rightarrow c + b = -\frac{c}{4}a \xrightarrow{c+b = -\frac{1}{c}} \boxed{a=1} \Rightarrow b + c = -\frac{1}{c} + c = \frac{c^2 - 1}{c} = -\frac{c}{4} \Rightarrow c^2 - 1 = -\frac{c}{4} \quad (5)$$

$$\Rightarrow c^2 + \frac{c}{4} - 1 = 0 \xrightarrow{a = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}} c_1 = \frac{-\frac{1}{4} + \sqrt{\frac{1}{16} + 4}}{2} = \frac{1}{4} \quad c_2 = \frac{-\frac{1}{4} - \sqrt{\frac{1}{16} + 4}}{2} = -2$$

$$c_1 = \frac{1}{4} \Rightarrow b = -\frac{1}{c} = -\frac{1}{\frac{1}{4}} = -4$$

$$c_2 = -2 \Rightarrow b = -\frac{1}{c} = -\frac{1}{-2} = \frac{1}{2}$$

$$(a+c)b = (1+\frac{1}{4})(-4) = \boxed{-5}$$

$$f(0) = \frac{2}{3} \Rightarrow 1 + cx^a = \frac{2}{3} \Rightarrow cx^a = -\frac{1}{3}$$

$$f(1) = 0 \Rightarrow 1 + cx^{a+b} = 0 \Rightarrow 1 + \frac{c}{-\frac{1}{3}} x^{a+b} = 0 \Rightarrow x^{a+b} = 3 \Rightarrow b = 1$$

$r > 1 \Rightarrow$

$$f(x) = 1 - r^{(x+1)} \Rightarrow f(-1) = 1 - r^{(-1+1)} = 1 - \frac{1}{r} = \frac{1}{a}$$

$$\Rightarrow c = -1, a = -1$$

$$f(x) = c + \log_\omega(ax+b)$$

$$f(0) = 2 \Rightarrow 2 = c + \log_\omega b$$

$$f(\frac{r}{r+1}) = 0 \Rightarrow 0 = c + \log_\omega(\frac{r}{r+1}a + b)$$

$$\Rightarrow \begin{cases} c + \log_\omega b = 2 \\ c - \log_\omega(\frac{r}{r+1}a + b) = 0 \end{cases} \Rightarrow \log_\omega b - \log_\omega(\frac{r}{r+1}a + b) = 2 \Rightarrow \log_\omega(\frac{b}{\frac{r}{r+1}a + b}) = 2 \quad (5)$$

$$r\omega = \frac{b}{\frac{r}{r+1}a + b} \Rightarrow \begin{cases} a + r\omega b = b \\ r\omega a = -r\omega b \end{cases} \Rightarrow \frac{a}{b} = -\frac{r\omega}{r\omega} = \boxed{-\frac{r}{\omega}}$$

$$|x^2 - 2| - x > 0 \Rightarrow \textcircled{1} \begin{cases} x > \sqrt{2} & x < -\sqrt{2} \\ x > 2 & x < -1 \end{cases} \Rightarrow x^2 - 2 - x > 0 \Rightarrow (x-2)(x+1) > 0$$

$$\Rightarrow x > 2 \text{ یا } x < -1$$

$$\textcircled{2} \begin{cases} x < \sqrt{2} \\ x < -\sqrt{2} \end{cases} \Rightarrow x^2 - 2 - x < 0 \Rightarrow (x-2)(x+1) < 0 \Rightarrow -1 < x < 2$$

$$D = [(-\infty, -1) \cup (2, +\infty)] \cup \left[ \frac{(-\infty, 1)}{(-\infty, \sqrt{2})} \cup (2, +\infty) \right]$$

$$g(1) = -1 + 3 + 1 = 1 \Rightarrow f(1) = 1 \Rightarrow r + r^{b-a} = 1 \Rightarrow \begin{cases} b-a = 3 \\ b+a = 3 \end{cases}$$

$$f^{-1}(1) = -1 \Rightarrow f(-1) = 1 \Rightarrow r + r^{b+a} = 1 \Rightarrow \begin{cases} b-a = 3 \\ b+a = 3 \end{cases} \Rightarrow a=0 \Rightarrow \frac{r^3}{10}$$

$$\Rightarrow rb - a = r(3) - 0 = \boxed{9}$$

$$\left. \begin{aligned} f(1) &= g(1) \rightarrow b - a = 1 \\ f^{-1}(1) &= -1 \rightarrow b + a = 3 \end{aligned} \right\} \Rightarrow \begin{aligned} a &= 1, b = 2 \\ rb - a &= 2 \end{aligned}$$

$$g(x) = x^r - x \Rightarrow g(1) = 0 \stackrel{f(1)=g(1)}{\Rightarrow} f(1) = 0 \Rightarrow -r + r^{-A-B} = 0 \Rightarrow \begin{cases} -A-B=1 \\ rA+B=-r \end{cases}$$

$$g(r) = r \stackrel{f(r)}{\Rightarrow} f(r) = r \Rightarrow -r + \left(\frac{1}{r}\right)^{A+B} = r \Rightarrow \begin{cases} rA+B=-r \\ A=-1 \Rightarrow B=0 \end{cases}$$

$$\Rightarrow f(x) = -r + \left(\frac{1}{r}\right)^{-x+0} = -r + r^x \Rightarrow f(r) = -r + r^r = -r + 1 = \boxed{6}$$

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$$\frac{1}{r} M_0 = M_0 \left(1 - \frac{1}{r}\right)^{\frac{t}{r_0}} \Rightarrow \frac{1}{r} = \left(\frac{1}{r}\right)^{\frac{t}{r_0}} \Rightarrow \frac{t}{r_0} = \log \frac{1}{\frac{1}{r}}$$

$$\log_r r = \frac{\log_r \Delta}{\log_r \omega} = \frac{\log_r \frac{1}{r}}{\log_r \frac{1}{r}} = \frac{r}{r}$$

$$\log \frac{1}{r} = \log \frac{1}{r} - (\log \frac{1}{r} + \log \frac{r}{r}) = -\left(\frac{1}{\log_r r - \log_r \omega} + \frac{1}{\log_r \frac{1}{r} - \log_r \frac{r}{r}}\right)$$

$$= -\left(\frac{1}{r - r \times \frac{r}{r}} + \frac{1}{r \times \frac{r}{r} - r}\right) = -\left(\frac{1}{-r} + \frac{1}{r}\right) = \frac{r}{r} + r = \frac{19}{r} \Rightarrow \frac{t}{r_0} = \frac{19}{r}$$

$$\Rightarrow t = r_0 \times 19 = \boxed{380 \text{ min}}$$

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$$\frac{1}{v} M_0 = M_0 \left(1 - \frac{v/\Delta}{100}\right)^{\frac{t}{v}} \Rightarrow \frac{1}{v} = \left(1 - \frac{v/\Delta}{100}\right)^{\frac{t}{v}} \Rightarrow \frac{t}{v} = \log \frac{1/v}{1 - v/\Delta}$$

$$\log_r r = \frac{\log_r r}{\log_r r} = \frac{\log_r \frac{1}{v}}{\log_r \frac{1}{v}} = \frac{r}{r}$$

$$\log \frac{1}{v} = \log \frac{1}{v} - \log \frac{v}{\Delta} = -\frac{1}{\log_r v - \log_r \Delta} = -\frac{1}{1 - r \log_r v}$$

$$= -\frac{1}{1 - r \times \frac{r}{r}} = -\frac{1}{1} = 1 \Rightarrow \frac{t}{v} = 1 \Rightarrow \boxed{t = 26 \text{ s}}$$

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$$\frac{1}{r} M_0 = M_0 \left(1 - \frac{r}{100}\right)^{\frac{t}{r}} \Rightarrow \frac{1}{r} = \left(\frac{r}{100}\right)^{\frac{t}{r}} \Rightarrow t = \log \frac{1/r}{r/100}$$

$$\log_r r = \frac{\log_r r}{\log_r r} = \frac{\log_r \frac{1}{r}}{\log_r \frac{r}{100}} = \frac{\Delta}{\lambda}$$

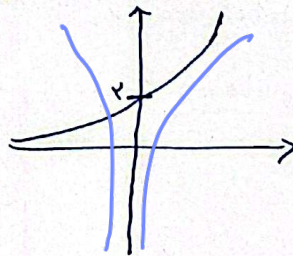
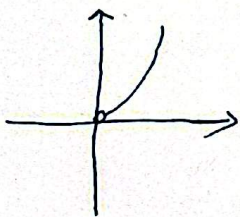
$$\log \frac{1}{r} = \log \frac{1}{r} - \log \frac{r}{100} = \frac{-1}{\log_r r - \log_r \omega} = \frac{-1}{1 + \log_r r - r \log_r \omega}$$

$$= \frac{-1}{1 + r \times \frac{\Delta}{\lambda} - r \times \frac{r \Delta}{r r}} = \frac{-1}{\frac{r r}{\lambda} - \frac{r \Delta}{r}} = \frac{-1}{\frac{r^2 - r \Delta}{r}} = \frac{-1}{r} = \boxed{24} \text{ s}$$

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الف)  $y = a^{\log_r x} = x^{\log_r a} = x^r$   $\hookrightarrow y = \log x^r = r \log x$   $D = ]R - \{0\}$

$D = (0, +\infty) \Rightarrow$



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