نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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$f(x)=1-\log_c^{(ax-b)} \Rightarrow f(0)=2 \Rightarrow 1-\log_c^{(-b)}=2 \Rightarrow \log_c^{(-b)}=-1 \Rightarrow -b=\frac{1}{c} \Rightarrow b=-\frac{1}{c}$

$f(-\frac{3}{2})=0 \Rightarrow 1-\log_c^{(-\frac{3}{2}a-b)}=0 \Rightarrow \log_c^{(-\frac{3}{2}a-b)}=1 \Rightarrow c=-\frac{3}{2}a-b$

$\Rightarrow c+b=-\frac{3}{2}a \xrightarrow[c+b=-\frac{3}{2}]{} a=1$ $\quad b+c=-\frac{1}{c}+c=\frac{c^2-1}{c}=-\frac{3}{2} \Rightarrow c^2-1=-\frac{3}{2}c$

$\Rightarrow c^2+\frac{3}{2}c-1=0 \xrightarrow[a=\frac{1}{2}+2=\frac{5}{2}]{} c_1=\frac{\frac{5}{2}-\frac{3}{2}}{2}=\frac{1}{2}$ قبول $\Rightarrow b=-\frac{1}{c}=-\frac{1}{\frac{1}{2}}=-2$

$c_2=\frac{-\frac{5}{2}-\frac{3}{2}}{2}=-2$ قبول $\dot{\in}(c>0, c\neq 1)$ $\quad (a+c)b=(1+\frac{1}{2})(-2)=\boxed{-3}$

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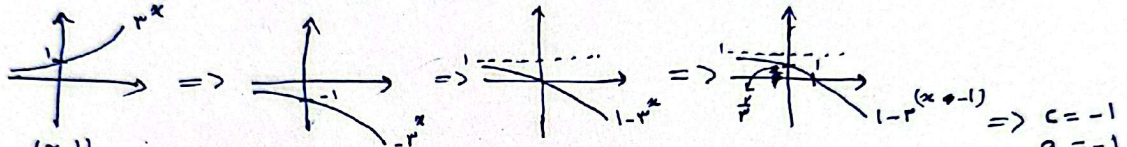
$f(0)=\frac{2}{3} \Rightarrow 1+c\times 3^a=\frac{2}{3} \Rightarrow c\times 3^a=-\frac{1}{3}$

$f(1)=0 \Rightarrow 1+c\times 3^{a+b}=0 \Rightarrow 1+\underbrace{c\times 3^a}_{-\frac{1}{3}}\times 3^b=0 \Rightarrow 3^b=3 \Rightarrow b=1$

$2>1 \Rightarrow$ 

$\Rightarrow c=-1$, $a=-1$

$f(x)=1-3^{(x-1)} \Rightarrow f(-1)=1-3^{(-1-1)}=1-\frac{1}{9}=\boxed{\frac{8}{9}}$

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$f(x)=c+\log_5^{(ax+b)}$

$f(0)=2 \Rightarrow 2=c+\log_5^b$

$f(2/4)=0 \Rightarrow 0=c+\log_5^{(2/4a+b)}$

$\Rightarrow \begin{cases} c+\log_5^b=2 \\ +\\ \underline{-c-\log_5^{(2/4a+b)}=0} \end{cases}$

$\log_5^b-\log_5^{(2/4a+b)}=2 \Rightarrow \log_5^{(\frac{b}{2/4a+b})}=2$

$2\times 5=\frac{b}{2/4a+b} \Rightarrow 60a+25b=b$

$60a=-24b \Rightarrow \frac{a}{b}=-\frac{24}{60}=\boxed{-\frac{2}{5}}$

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$|x^2-2|-x>0 \Rightarrow$ ① اگر $x^2-2-x>0 \Rightarrow (x-2)(x+1)>0$

$\Rightarrow x>2$ یا $x<-1 \xrightarrow[\text{فرض اولیه}]{\text{اشتراک با}} x>2$ یا $x<-1$ (فرض اولیه: $x>\sqrt{2}$ یا $x<-\sqrt{2}$)

② اگر $-\sqrt{2}<x<\sqrt{2}$ (فرض اولیه) $\Rightarrow x^2-2-x<0 \Rightarrow (x-2)(x+1)<0 \Rightarrow -1<x<2 \xrightarrow[\text{فرض اولیه}]{\text{اشتراک با}} -\sqrt{2}<x<\sqrt{2}$

$D=[(-\infty,-1)\cup(2,+\infty)]\cup(-\sqrt{2},\sqrt{2}) = \boxed{(-\infty,\sqrt{2})\cup(2,+\infty)}$

۵

$g(1)=-1+3+8=10 \xrightarrow[f(1)=g(1)]{} f(1)=10 \Rightarrow 2+2^{b-a}=10 \Rightarrow \begin{cases} b-a=3 \\ +\\ \underline{b+a=3} \end{cases}$

$f^{-1}(10)=-1 \Rightarrow f(-1)=10 \Rightarrow 2+2^{b+a}=10 \Rightarrow b=2 \Rightarrow a=0$ تابع ثابت ۱۵

$\Rightarrow 2b-a=2(3)-0=\boxed{6}$

$g(x)=x^2-x \Rightarrow g(1)=0 \overset{f(1)=g(1)}{\Rightarrow} f(1)=0 \Rightarrow -2+2^{-A-B}=0 \Rightarrow -A-B=1$

$g(2)=2 \Rightarrow f(2)=2 \Rightarrow -2+\left(\frac{1}{2}\right)^{2A+B}=2 \Rightarrow 2A+B=-2$

$A=-1 \Rightarrow B=0$

$\Rightarrow f(x)=-2+\left(\frac{1}{2}\right)^{-x+0}=-2+2^x \Rightarrow f(3)=-2+2^3=-2+8=6$ ۶

$\frac{1}{6}m_0=m_0\left(1-\frac{1}{9}\right)^{\frac{t}{60}} \Rightarrow \frac{1}{6}=\left(\frac{8}{9}\right)^{\frac{t}{60}} \Rightarrow \frac{t}{60}=\log_{\frac{8}{9}}\frac{1}{6}$

$\log_3^2=\frac{\log 2}{\log 3}=\frac{0.3}{0.48}=\frac{5}{8}$

$\log_{\frac{8}{9}}\frac{1}{6}=\log_{\frac{8}{9}}1-\left(\log_{\frac{8}{9}}2+\log_{\frac{8}{9}}3\right)=-\left(\frac{1}{\log_2^{\frac{8}{9}}}+\frac{1}{\log_3^{\frac{8}{9}}}\right)$

$=-\left(\frac{1}{3-2\log_2^3}+\frac{1}{2\log_3^2-2}\right)=-\left(\frac{1}{-\frac{1}{4}}+\frac{1}{-\frac{1}{4}}\right)=\frac{8}{3}+2=\frac{14}{3} \Rightarrow \frac{t}{60}=\frac{14}{3}$

$\Rightarrow t=20\times 14=280\ \text{min}$ ۷

$\frac{1}{8}m_0=m_0\left(1-\frac{12.5}{100}\right)^{\frac{t}{4}} \Rightarrow \frac{1}{8}=\left(\frac{7}{8}\right)^{\frac{t}{4}} \Rightarrow \frac{t}{4}=\log_{\frac{7}{8}}\frac{1}{8}$

$\log_7^2=\frac{\log 2}{\log 7}=\frac{0.3}{0.84}=\frac{3}{8}$

$\log_{\frac{7}{8}}\frac{1}{8}=\log_{\frac{7}{8}}1-\log_{\frac{7}{8}}8=-\frac{1}{\log_7^8-\log_8^8}=-\frac{1}{3\log_7^2-1}$

$=-\frac{1}{3\times\frac{3}{8}-1}=-\frac{1}{-\frac{1}{8}}=8 \Rightarrow \frac{t}{4}=8 \Rightarrow t=32$ روز ۸

$\frac{1}{3}m_0=m_0\left(1-\frac{5}{100}\right)^{\frac{t}{2}} \Rightarrow \frac{1}{3}=\left(\frac{95}{100}\right)^{\frac{t}{2}} \Rightarrow \frac{t}{2}=\log_{\frac{95}{100}}\frac{1}{3}$

$\log_3^2=\frac{\log 2}{\log 3}=\frac{0.3}{0.48}=\frac{5}{8}$

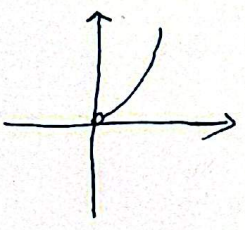
$\log_3^5=\log_3^{10}-\log_3^2=\frac{1}{\log 3}-\log_3^2=\frac{100}{48}-\frac{5}{8}=\frac{35}{24}$

$t=\log_{\frac{95}{100}}\frac{1}{3}=\log_{\frac{95}{100}}1-\log_{\frac{95}{100}}3=\frac{-1}{\log_3^{95}-\log_3^{100}}=\frac{-1}{1+2\log_3^5-2\log_3^2}$

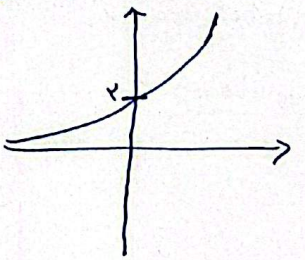
$=\frac{-1}{1+2\times\frac{35}{24}-2\times\frac{5}{8}}=\frac{-1}{\frac{24}{12}-\frac{15}{12}}=\frac{-1}{\frac{9}{24}}=\frac{-1}{\frac{3}{8}}=-\frac{8}{3}$ روز ۹

الف) $y=9^{\log_3^2}=x^{\log_3^9}=x^2$

$D=(0,+\infty) \Rightarrow$



ب) $y=\log x^2=2\log x$



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