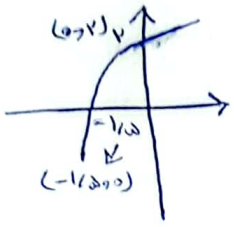
نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

۱

$y = 1 - \log_c^{(ax-b)} + bc = -\frac{3}{2}$ $\quad (a+c)b = 5$



$\Rightarrow 0 = 1 - \log_c^{(-\frac{1}{2}a+b)} \Rightarrow 1 = \log_c^{(-\frac{1}{2}a-b)} \Rightarrow -\frac{1}{2}a - b = c^1$

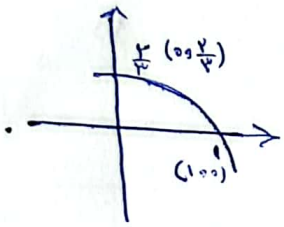
$a=1$ $\quad 2 = 1 - \log_c^{(-b)} \Rightarrow -1 = \log_c^{-b} \Rightarrow -b = c^{-1}$

$bc=-1 \Rightarrow (-\frac{3}{2}-c)c = -1 \Rightarrow c^2 + \frac{3}{2}c - 1 = 0 \Rightarrow c = -2 \text{ یا } c = \frac{1}{2}$

$bc = -1 \Rightarrow b \times \frac{1}{2} = -1 \Rightarrow b = -2$ $\quad (1+\frac{1}{2}) \times -2 = -3$

۲

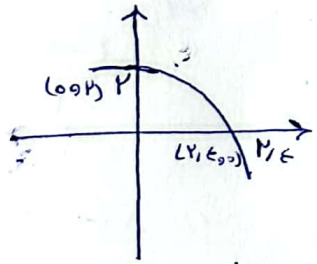
$F(x) = 1 + c \times 3^{ax+b}$ $\quad f(-1) = 5$



$\begin{cases} \frac{2}{3} = 1 + c \times 3^a \\ 0 = 1 + c \times 3^{a+b} \end{cases} \Rightarrow \begin{cases} -\frac{1}{3} = c \times 3^a \\ -1 = c \times 3^a \times 3^b \end{cases}$

۳

$y = c + \log_a^{(ax+b)}$ $\quad \frac{a}{b} = 5$



$\begin{cases} c + \log_a^b = 2 \\ c + \log_a^{2x+4a+b} = 0 \end{cases}$

$\log_a^{2x+4a+b} - \log_a^b = -2$

$\log_a \frac{2x+4a+b}{b} = -2$

$\frac{2x+4a+b}{b} = \frac{1}{25}$

$\frac{2x+4a}{b} + 1 = \frac{1}{25} \Rightarrow \frac{2x+4a}{b} = -\frac{24}{25}$

$\frac{a}{b} = -\frac{24}{25} \times \frac{1}{24} = -\frac{1}{25} = -\frac{2}{5}$

۴

$f(x) = \log_{\frac{1}{2}}^{(|x^2-4|-x)}$ $\quad D_f = (-\infty, -1) \cup (4, +\infty)$

$|x^2-4|-x > 0 \Rightarrow |x^2-4| > x$ (۱)

$x^2 - 4 \geq 0 \Rightarrow x \leq -2 \text{ یا } x \geq 2$

$x^2 - x - 4 > 0 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x = 2 \text{ یا } x = -1 \Rightarrow x < -1 \text{ یا } x > 2$

$x \in (-\infty, -2] \cup (2, +\infty)$ (۲) $\quad x^2 - 4 < 0 \Rightarrow -2 < x < 2 \Rightarrow x^2 + x - 2 > 0$

$x = -2 \text{ یا } x = 1$

$\Rightarrow ((-\infty, -2] \cup (2, +\infty)) \cap (-2, 1)$

۵

$\begin{cases} y = x^2 - x \xrightarrow{x=1} y = 0 \\ y = x^2 - x \xrightarrow{x=2} y = 2 \end{cases} \Rightarrow \begin{cases} (1,0) \\ (2,2) \end{cases} \Rightarrow \begin{cases} f(1) = 0 \\ f(2) = 2 \end{cases} \Rightarrow \begin{cases} -2 + (\frac{1}{2})^{A+B} = 0 \\ -2 + (\frac{1}{2})^{A+B} = 2 \end{cases}$

$\Rightarrow \begin{cases} (2^{-1})^{A+B} = 2^1 \\ (2^{-1})^{2A+B} = 2^2 \end{cases} \Rightarrow \begin{cases} -(A+B) = 1 \\ -(2A+B) = 2 \end{cases} \Rightarrow A = -1, B = 0.$

سوال ۵ جا به جا

$f(x) = -2 + (\frac{1}{2})^{-x} \Rightarrow -2 + (\frac{1}{2})^{-3} \Rightarrow -2 + 2^3 = 6$

$f(x)=2+2^{b-ax}$, $g(x)=-x^2-3x+8$ $(1,0)$, $f^{-1}(10)=-1$, $2b-a=?$

$f(1)=g(1) \Rightarrow 2+2^{b-a}=-1-3+8 \Rightarrow 2^{b-a}=2 \Rightarrow b-a=1$

$f(-1)=10 \Rightarrow f(-1)=2+2^{b-a(-1)}=10 \Rightarrow 2^{b+a}=8 \Rightarrow b+a=3$ ۶

$\begin{cases} b-a=1 \\ b+a=3 \end{cases} \Rightarrow a=1 , b=2$ $2\times 2-1=3$ سوال ۶ جا به جا

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۳۸۰ min

$m_1 \Rightarrow m(1)=m_1-\frac{1}{9}m_1=\left(\frac{8}{9}\right)m_1$ (یک ساعت) جرم اولیه

$m(2)=m(1)-\frac{1}{9}m(1)=\left(\frac{8}{9}\right)m(1)=\left(\frac{8}{9}\right)^2 m_1$ (دو ساعت) $m(t)=\frac{1}{4}m_1 \Rightarrow \left(\frac{8}{9}\right)^t=4^{-1}$

$m(t)=m_1\left(\frac{8}{9}\right)^t \Rightarrow \log_5 \left(\frac{8}{9}\right)^t=\log_5 4^{-1} \Rightarrow t(\log_5 8-\log_5 9)=-\log_5 4$ ۷

$\log_5 2=\frac{1}{\log_2 5} \Rightarrow \frac{5}{12}$, $\log_5 3=\frac{1}{\log_3 5}=\frac{5}{7}$

$t=\dfrac{\frac{5}{12}+\frac{5}{7}}{\frac{10}{7}-\frac{15}{12}}=\frac{95}{15}=\frac{19}{3}$ ساعت $\Rightarrow \frac{19}{3}\times 60=380$

$100-12.5=87.5$ $m(t)=m_1\left(\frac{87.5}{100}\right)^t \Rightarrow m(t)=\frac{1}{7}m_1$

$\left(\frac{875}{1000}\right)^t=\frac{1}{7} \Rightarrow \left(\frac{7\times 125}{1000}\right)^t=\frac{1}{7} \Rightarrow \left(\frac{7}{8}\right)^t=\frac{1}{7}$ $\xrightarrow{\text{در آخر}} t=\dfrac{-1}{1-\log_7 8}$ ۸

$\log_7 \left(\frac{7}{8}\right)^t=\log_7 \frac{1}{7} \Rightarrow t\log_7 \frac{7}{8}=\log_7 7^{-1} \Rightarrow t=\dfrac{-1}{1-\log_7 8}$ $\dfrac{-1}{1-\frac{9}{8}}=8$

$\log_7 2^3=3\log_7 2=3\times\dfrac{\log_3 2}{\log_3 7} \Rightarrow \frac{9}{8} \Rightarrow$ ۵۶ روز $=8\times 7$

غلظت $=\frac{m}{100}$ (روز) $\Rightarrow$ روز اول $\Rightarrow \frac{6}{100} \Rightarrow m-0.06m=0.94m$ (غلظت کل)

$\frac{m}{100}(0.94)^n$ (روز nام) $\Rightarrow \frac{1}{3}\times\frac{m}{100} \Rightarrow (0.94)^n=\frac{1}{3} \Rightarrow n\log 0.94=\log\left(\frac{1}{3}\right)$ ۹

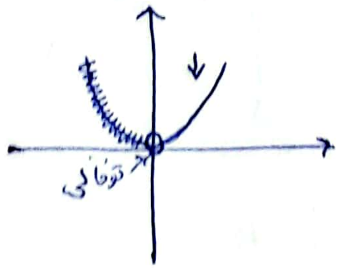
$\Rightarrow n=\dfrac{\log 3^{-1}}{\log 0.94}=\dfrac{-\log 3}{\log 94-\log 100}=\dfrac{-0.48}{0.48+5(0.3)-2}=\dfrac{+0.48}{+0.02}=24$ روز

$\log(3\times 2^5) \rightarrow \log 94$; $\log 3+5\log 2$

الف) $y=9^{\log_3 x} \Rightarrow y=x^{\log_3 9}=x^2$

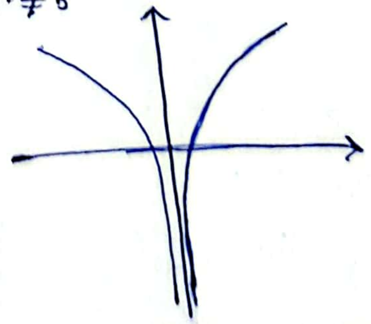
$x>0$

$x\neq 0$



ب) $y=\log x^2 \Rightarrow y=2\log x$

$x^2>0$ $x^2\neq 0$



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