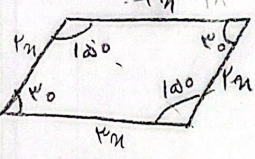


مستطای الاضلاع به مساحت ۵۴، نسبت دو ضلع مجاور آن به ۳ است. زاویه بزرگترین بین دو ضلع مجاور ۱۵۰°، مساحت آن چقدر است؟



$$\sin 150^\circ = \sin 180^\circ - 30^\circ = \frac{1}{2}$$

$$S = \frac{1}{2} \times p \times m \times \sin 150^\circ = 54$$

$$2^2 = 1^2 + m^2 \Rightarrow m = \sqrt{11}$$

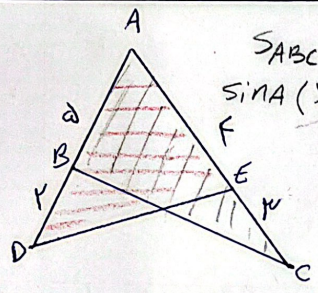
$$p = 2\sqrt{11} = 4\sqrt{11}$$

$$m = 2\sqrt{11} = 4\sqrt{11}$$

$$P_{\text{بزرگ}} = \frac{1}{2} (4\sqrt{11} + 4\sqrt{11}) \times 4\sqrt{11} = 176$$

۱

اختلاف مساحت ABC و ADE برابر ۱۷۵ است.  $\tan A = ?$



$$S_{ABC} - S_{ADE} = \sin A \left( \frac{1}{2} \times v \times u - \frac{1}{2} \times \frac{v}{2} \times \frac{u}{2} \right) = \frac{v}{2}$$

$$\sin A \left( \frac{vu}{2} - \frac{vu}{8} \right) = \frac{v}{2} \Rightarrow \frac{3vu \sin A}{8} = \frac{v}{2} \Rightarrow \sin A = \frac{4}{3}$$

$$\rightarrow 1 + \cot^2 A = \frac{1}{\sin^2 A} \rightarrow \cot^2 A - 1 = 2$$

$$\cot A = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

۲

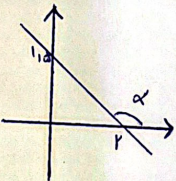
انتقالی در کمانها:

$$\frac{|\sin \alpha|}{\cos \alpha} = \frac{1}{\cot \alpha} \Rightarrow \frac{1}{\sqrt{\cos \alpha}} - \tan \alpha = \frac{1 + \sin \alpha}{\cos \alpha}$$

$$\frac{|\sin \alpha|}{\cos \alpha} = -\frac{\sin \alpha}{\cos \alpha} \rightarrow \sin \alpha < 0 \rightarrow \frac{1}{\cos \alpha} - \tan \alpha = \frac{1}{\cos \alpha} + \frac{\sin \alpha}{\cos \alpha} = \frac{1 + \sin \alpha}{\cos \alpha} \rightarrow \cos \alpha < 0$$

۳

$a \sin \alpha + b$



$$(0, b) \rightarrow b = \frac{p}{r}$$

$$(r, 0) \rightarrow ra + \frac{p}{r} = 0 \Rightarrow ra = -\frac{p}{r} \Rightarrow a = -\frac{p}{r} = \tan \alpha$$

$$\tan \left( \frac{\pi}{4} - \alpha \right) = \cot \alpha = \frac{-\frac{p}{r}}{\frac{p}{r}} = -1$$

۴

$$\frac{r \cos(170^\circ) - r \sin(120^\circ)}{\sin(170^\circ) - \cos(120^\circ)} = \frac{r \cos\left(\frac{17\pi}{18}\right) - r \sin\left(\frac{2\pi}{3}\right)}{\sin\left(\frac{17\pi}{18}\right) - \cos\left(\frac{2\pi}{3}\right)}$$

$$= \frac{-r \sin - r \sin}{-\sin - \sin} = \frac{-2r \sin}{-2 \sin} = \frac{r \sin}{\sin} = r \sin$$

۵

$\frac{\sin(\frac{\pi}{r} + \alpha) - \sin(\alpha - \pi)}{|\tan \alpha - 1|} =$   
 $\frac{\cos \alpha + \sin \alpha}{|\frac{1}{\cos r} - r|} = \frac{\frac{r}{r} - \frac{\sqrt{a}}{r}}{|\frac{1}{r} - \frac{1}{r}|} = \frac{r - \sqrt{a}}{r} = \frac{1 - r\sqrt{a}}{r}$

$\cos^2 + \sin^2 = 1$   
 $\cos \alpha = \frac{r}{r}$   
 $\sin \alpha = \frac{\sqrt{a}}{r}$

6

$\sin \alpha = r \cos \alpha \rightarrow \frac{\sin \alpha}{\cos \alpha} = r = \tan \alpha$   
 $1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha} \rightarrow 1 + r^2 = \frac{1}{\cos^2 \alpha}$

$\cos \alpha = \frac{1}{\sqrt{1+r^2}}$   
 $|\cos \alpha| = \frac{1}{\sqrt{a}}$   
 $\frac{1}{\sqrt{a}} = \cos \alpha$

7

$\tan 40^\circ = \sqrt{r} = \frac{m}{m^2 - 1}$   
 $\sqrt{r} m^2 - \sqrt{r} + r m = 0 \rightarrow \sqrt{r} m^2 + r m - \sqrt{r} = 0$   
 $m^2 + r m - 1 = 0$   
 $(m+1)(m-1) = 0$   
 $m = -\frac{r}{\sqrt{r}} = -\frac{\sqrt{r}}{r}$   
 $m = \frac{1}{\sqrt{r}} = \frac{\sqrt{r}}{r}$

$m_1 - m_2 = \frac{\sqrt{r}}{r} + \sqrt{r} = \frac{\sqrt{r} + r\sqrt{r}}{r} = \frac{r\sqrt{r}}{r}$

8

$-\frac{\pi}{r} < \alpha < \frac{\pi}{r} \rightarrow -\frac{\pi}{r} < -\alpha < \frac{\pi}{r}$   
 $0 < \tan(\frac{\pi}{r} - \alpha) < 1$   
 $0 < \frac{1-m}{r+m} < 1$   
 $\frac{1-m}{r+m} = \frac{-r}{r+1}$

9

$\tan(40^\circ) \cos(110^\circ) + \tan(110^\circ) \sin(40^\circ) =$   
 $\frac{-\sqrt{r}}{r} \times \frac{-\sqrt{r}}{r} + \frac{-\sqrt{r}}{r} \times \frac{\sqrt{r}}{r} =$   
 $\frac{r}{r} - \frac{r}{r} = 0$

$110^\circ = 40^\circ + 70^\circ$   
 $140^\circ = 70^\circ + 70^\circ$

10