آرتیمیس اکبری

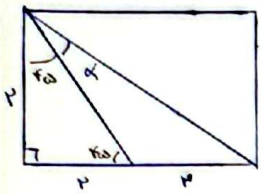
سوال ① =>

$S_{مثلث} = \frac{1}{2} ab\sin\alpha \rightarrow 4\sqrt{3} = \frac{1}{2} \times 6 \times 5\sqrt{3} \times \sin\alpha \rightarrow \sin\alpha = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \rightarrow \alpha \begin{cases} 120^\circ \\ 60^\circ \end{cases}$

$\frac{\text{بیشترین مقدار } \alpha}{\text{کمترین مقدار } \alpha} = \frac{120}{60} = 2$ ✓

سوال ② =>

$\star \cot(\alpha \pm \beta) = \frac{\cot\alpha\cot\beta \mp 1}{\cot\alpha \pm \cot\beta}$



$\cot\alpha = ?$

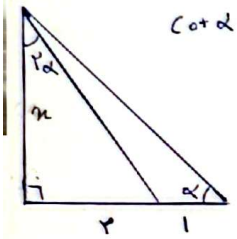
$\cot(\alpha + 45) = \frac{2}{4} = \frac{1}{2}$

$\cot(\alpha+45) = \frac{\cot\alpha\cot 45 - 1}{\cot\alpha + \cot 45} = \frac{\cot\alpha \times 1 - 1}{\cot\alpha + 1} = \frac{1}{2}$

$\rightarrow 2\cot\alpha - 2 = \cot\alpha + 1$

$\cot\alpha = 3$ ✓

سوال ③ =>



$\cot\alpha = ?$

$\tan 2\alpha = \frac{2}{x}$

$\tan\alpha = \frac{x}{3}$

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

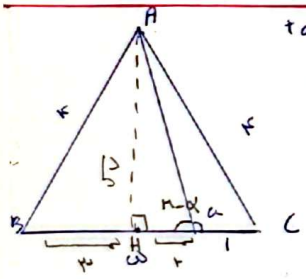
$\rightarrow \frac{2(\frac{x}{3})}{1-(\frac{x}{3})^2} = \frac{2}{x} \rightarrow \frac{\frac{2x}{3}}{\frac{9-x^2}{9}} = \frac{2}{x} \rightarrow \frac{6x}{9-x^2} = \frac{2}{x}$

$6x^2 = 18 - 2x^2 \rightarrow 8x^2 = 18 \rightarrow x^2 = \frac{9}{4} \rightarrow x = \frac{3}{2}$

چون x مقدار منفی است نمی‌تواند منفی باشد

$\cot\alpha = \frac{3}{x} = \frac{3}{\frac{3}{2}} = 2$ ✓

سوال ④ =>



$\tan\alpha = ?$

میانه ضلع ارتفاع است ← مثلث متساوی‌الساقین

$AH = \sqrt{19-9} = \sqrt{10}$

$\tan(\pi-\alpha) = \frac{\sqrt{10}}{2} \rightarrow -\tan\alpha = \frac{\sqrt{10}}{2}$

$\tan\alpha = \frac{-\sqrt{10}}{2}$ ✓

سوال ⑤ =>

$2\sin^2 x + \cos^2 x = \frac{4}{3}$, $\tan^2 x = ?$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x + \sin^2 x = \frac{4}{3} - 1 \rightarrow \sin^2 x = \frac{1}{3} \rightarrow \sin x = \begin{cases} +\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{cases}$

$\cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{3} = \frac{2}{3}$

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$ ✓

سوال ⑥ =>

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha}$$

$$= \frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{2-\sin^2\alpha} - \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{2-\cos^2\alpha} = \frac{\sin^4\alpha - 4\sin^2\alpha + 4}{2-\sin^2\alpha} - \frac{\cos^4\alpha - 4\cos^2\alpha + 4}{2-\cos^2\alpha} = \frac{(\sin^2\alpha - 2)^2}{2-\sin^2\alpha} - \frac{(\cos^2\alpha - 2)^2}{2-\cos^2\alpha}$$

$$= (2-\sin^2\alpha) - (2-\cos^2\alpha) \quad \cos^2\alpha - \sin^2\alpha = \boxed{\cos 2\alpha}$$

⑥

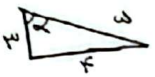
سوال ⑦ =>

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{11\pi}{2}\right) = ?$$

$$= -\tan\left(\frac{11\pi}{2} - \alpha\right)$$

$$= \cos\alpha \times (-\sin\alpha) \cdot \cot\alpha$$

$\tan\alpha = \frac{4}{3}$ ، α در ربع سوم



$\sin, \cos < 0$

$\tan, \cot > 0$

$$= -\frac{3}{5} \times \frac{4}{5} + \frac{3}{4} = \frac{-12}{25} + \frac{3}{4} = \frac{-48+75}{100} = \boxed{\frac{27}{100}}$$

⑦

سوال ⑧ =>

$$3\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x \quad , x = \frac{\pi}{12}$$

$$= 3\cos 4x + \sqrt{2}(\sin x - \cos x) \qquad \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$$

$$= 3\cos 4\left(\frac{\pi}{12}\right) + 2\sin\left(\frac{\pi}{12} - \frac{\pi}{4}\right) = 3\cos\frac{\pi}{3} + 2\sin\left(-\frac{\pi}{6}\right)$$

$$= \frac{3}{2} - 1 = \boxed{\frac{1}{2}}$$

⑧

سوال ⑨ =>

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = ? \quad , \quad \tan\left(\frac{\alpha}{2}\right) = \frac{1}{4}$$

$$\star \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \qquad \star \sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}$$

$$\star \cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$$

$$\tan\alpha = \tan 2\left(\frac{\alpha}{2}\right) = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{16}{30} = \frac{8}{15}$$

$$\cos\alpha = \cos 2\left(\frac{\alpha}{2}\right) = \frac{1-\tan^2\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{\frac{15}{16}}{\frac{17}{16}} \cdot \frac{15}{17}$$

$$\sin\alpha = \sin 2\left(\frac{\alpha}{2}\right) = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{\frac{1}{2}}{\frac{17}{16}} \cdot \frac{8}{17}$$

$$\rightarrow \frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \boxed{\frac{-16}{105}}$$

⑨

سوال ⑩ =>

$2\sin\alpha < \sin 2\alpha$ ، $0 < \frac{\cot\alpha}{\sin\alpha}$ ؛ α در ربع

$$\frac{\cot\alpha}{\sin\alpha} > 0 \rightarrow \frac{\cos\alpha}{\sin^2\alpha} > 0 \rightarrow \cos\alpha > 0 \quad \text{I}$$

$\sin^2\alpha$ ⊕

$I, II \rightarrow$ ربع چهارم

⑩

$$2\sin\alpha < \sin 2\alpha \rightarrow \cancel{2}\sin\alpha < \cancel{2}\sin\alpha\cos\alpha \rightarrow \sin\alpha\cos\alpha - \sin\alpha > 0$$

$$\sin\alpha(\cos\alpha - 1) > 0 \rightarrow \sin\alpha < 0 \quad \text{II}$$

$\cos\alpha - 1$ ⊖