تکلیف شماره ۲۷ — یازدهم دفتر B

Date: / / — Sat Sun Mon Tue Thu Wed Fri — Subject: حسابان

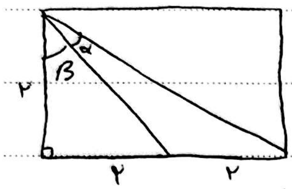
۲۰

۱)

$$S = \tfrac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha = 4.5 \to \sin\alpha = \frac{4.5}{3\sqrt{3}} = \frac{\sqrt{3}}{2} \to \alpha = 60 \text{ یا } 120^\circ$$

$$\boxed{\frac{120}{60} = 2}$$

۲)

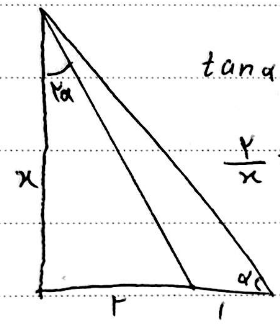


$$\tan\beta = 1 \qquad \tan(\alpha+\beta) = 2$$

$$\tan\alpha = \tan((\alpha+\beta)-\beta) = \frac{\tan(\alpha+\beta) - \tan\beta}{\tan(\alpha+\beta)\tan\beta + 1} = \frac{2-1}{2+1} = \frac{1}{3}$$

$$\to \boxed{\cot\alpha = \frac{1}{1/3} = 3}$$

۳)

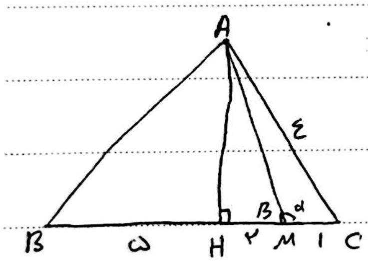


$$\tan\alpha = \frac{x}{3} \qquad \tan 2\alpha = \frac{2}{x} \to \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \to \frac{2}{x} = \frac{\frac{2x}{3}}{1-\frac{x^2}{9}} \to$$

$$\frac{2}{x} = \frac{6x}{9-x^2} \to 6x^2 = 18 - 2x^2 \to 18 = 8x^2 \to x^2 = \frac{9}{4} \to x = \pm\frac{3}{2} \to x = \frac{3}{2}$$

$$\tan\alpha = \frac{x}{3} = \frac{3/2}{3/1} = \frac{1}{2} \to \boxed{\cot\alpha = \frac{1}{1/2} = 2}$$

۴)



$$HC = \frac{6}{2} = 3 \to HM = 2 \to AH = \sqrt{16-9} = \sqrt{7}$$

$$\tan\beta = \frac{AH}{HM} = \frac{\sqrt{7}}{2} \to \tan\alpha = \tan(180-\beta) = \boxed{\frac{-\sqrt{7}}{2}}$$

$-\tan\beta$

۵)

$$\sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \frac{4}{3} \to \sin^2 x = \frac{1}{3} \Rightarrow \cos^2 x = \frac{2}{3} \to \tan^2 x = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{\frac{1}{2}}$$

۶)

$$\frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{1+\sin^2\alpha} \Rightarrow \frac{(\sin^2\alpha - 2)^2}{1+\cos^2\alpha} - \frac{(\cos^2\alpha-2)^2}{1+\sin^2\alpha}$$

$$= \frac{(-1-\cos^2\alpha)^2}{1+\cos^2\alpha} - \frac{(-1-\sin^2\alpha)^2}{1+\sin^2\alpha} = 1+\cos^2\alpha - (+1+\sin^2\alpha) = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$$

۷)

$$\sin\left(\tfrac{9\pi}{2}+\alpha\right) \to \cos\alpha$$

$$\cos\left(\tfrac{7\pi}{2}-\alpha\right) \to -\sin\alpha$$

$$-\tan\left(\alpha-\tfrac{9\pi}{2}\right) = \tan\left(\tfrac{3\pi}{2}-\alpha\right) = \cot\alpha$$

$$\tan\alpha = \frac{4}{3} \to \sin = -\frac{4}{5},\ \cos = -\frac{3}{5}$$

$$\Rightarrow \cos\alpha(-\sin\alpha) + \cot\alpha \to \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) + \frac{3}{4} =$$

$$-\frac{12}{25} + \frac{3}{4} = \frac{-48+75}{100} = \frac{27}{100} = 0.27$$

(۸)

$$3\cos 4x + 2\left(\frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x\right) = 3\cos 4x + 2\sin\left(x - \frac{\pi}{4}\right)$$

$$\Rightarrow 3\cos\frac{\pi}{3} + 2\sin\left(-\frac{\pi}{4}\right) = 3\left(\frac{1}{2}\right) - 2\left(\frac{1}{2}\right) = \boxed{\frac{1}{2}}$$

(۲)

(۹)

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{\frac{1}{2}}{1-\frac{1}{16}} = \frac{8}{15}, \quad \sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{\frac{1}{2}}{1+\frac{1}{16}} = \frac{8}{17} \rightarrow \cos\alpha = \frac{15}{17}$$

(۴)

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{136-120}{255}}{\frac{-7}{17}} = \frac{16\times 17}{-7\times 255} = -\frac{16}{105}$$

(۱۰)

$$2\sin\alpha < 2\sin\alpha\cos\alpha \longrightarrow \sin\alpha - \sin\alpha\cos\alpha < 0 \rightarrow \sin\overbrace{(1-\cos\alpha)}^{+} < 0$$

$$\Rightarrow \sin\alpha < 0, \quad \frac{\cot\alpha}{\sin\alpha} > 0 \xrightarrow{\sin\alpha < 0} \cot\alpha < 0$$

(۳)

$$\begin{cases}\cot < 0 \\ \sin < 0\end{cases} \longrightarrow \cos\alpha > 0 \longrightarrow \begin{cases}\cos\alpha > 0 \\ \sin\alpha < 0\end{cases}$$ ربع چهارم