Date ۲۷ تکلیف

آنیا کریمی یازدهم دفتر B Subject

۱۸/۵

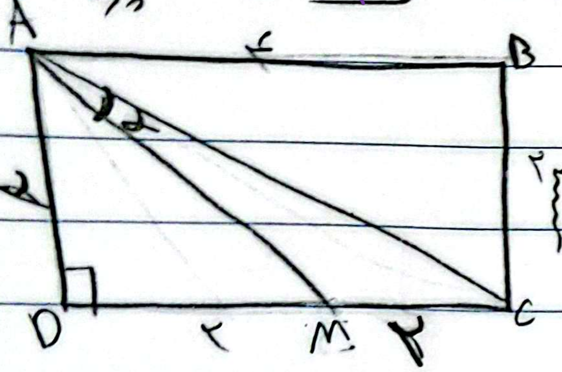
۱) ۱/۵

$\frac{1}{2}\times\sqrt{3}\times 6\times\sin\alpha = ۴/۵ =$

$\rightarrow 3\sqrt{3}\sin\alpha = \frac{9}{2} \Rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$ $\begin{cases} min = 60 \\ max = ~~60~~ \; 120 \end{cases}$

~~$\frac{3}{60} = 5$~~ $\frac{120}{60} = 2$

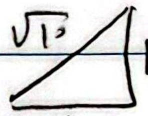
۲) ۵



$AM = \sqrt{2^2+2^2} = 2\sqrt{2}$

$AC = \sqrt{2^2+4^2} = \sqrt{20} = 2\sqrt{5}$

$\cos\alpha = \frac{3}{\sqrt{10}}$



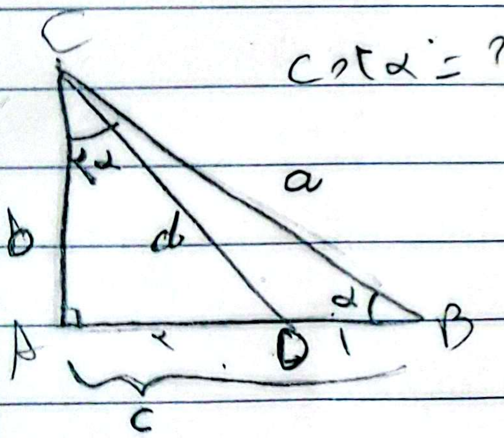
$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\frac{3}{\sqrt{10}}}{\frac{1}{\sqrt{10}}} = 3$

$S_{AMC} = S$ مستطیل − مساحت دو مثلث $= 8 - (2+4) = 2$

$2 = \frac{1}{2}\times 2\sqrt{5}\times 2\sqrt{2}\times\sin\alpha \Rightarrow \sin\alpha = \frac{1}{\sqrt{10}}$

۳) ۵

$\cot\alpha = ?$



$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \frac{b}{d} = \left(\frac{3}{a}\right)^2 - \left(\frac{b}{a}\right)^2 \Rightarrow \frac{9-b^2}{a^2}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\frac{2}{d} = 2\left(\frac{b}{a}\right)\left(\frac{3}{a}\right) = \frac{6b}{a^2} \Rightarrow a^2 = 3ad$

$\rightarrow \frac{9-b^2}{a^2} = \frac{9-b^2}{3ad} = \frac{b}{d} \Rightarrow b = \frac{3}{2}$

$\cot\alpha = \frac{3}{(b)} \Rightarrow \frac{3}{\frac{3}{2}} = 2$

$2\sin^2 x + \cos^2 x = \frac{4}{3}$ $\quad \tan^2 x = ?$ ④

$2(1-\cos^2 x) + \cos^2 x = \frac{4}{3}$

$2 - 2\cos^2 x + \cos^2 x = \frac{4}{3} \cdot \quad 2 - \cos^2 x = \frac{4}{3}$ ۲

$\cos^2 x = \frac{2}{3}$

$\sin^2 x = 1 - \cos^2 x = 1 - \frac{2}{3} = \frac{1}{3}$

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{\frac{1}{2}}$

⑤

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha}$$

$$\frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{1+\sin^2\alpha}$$

۶

$$\frac{\sin^4\alpha + 4 - 4\sin^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4 - 4\cos^2\alpha}{1+\sin^2\alpha}$$

$$\frac{(\sin^2\alpha - 2)^2}{\underbrace{1+\cos^2\alpha}_{2-\sin^2\alpha}} - \frac{(\cos^2\alpha - 2)^2}{\underbrace{1+\sin^2\alpha}_{2-\cos^2\alpha}} = (2-\sin^2\alpha) - (2-\cos^2\alpha)$$

$$= \cos 2\alpha$$

⑦

$\tan\alpha = \frac{4}{3} \qquad \frac{1}{\cos^2\alpha} = 1 + \tan^2\alpha \Rightarrow 1 + \frac{16}{9} = \frac{25}{9}$

$\Rightarrow \cos^2\alpha = \frac{9}{25} \xrightarrow{\alpha \in (\pi, \frac{3\pi}{2})} \cos\alpha = \frac{-3}{5}$

۱

$$\sin\left(\frac{3\pi}{2} + \alpha\right)\cos\left(\frac{7\pi}{2} - \alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right)$$

$$= (+\cos\alpha)(-\cos\alpha) + \cot\alpha$$

$$\frac{-1}{5} + \frac{3}{4} = \frac{-21}{20}$$

POODLE

$x = \frac{\pi}{12}$ ①

$(2\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x)$

$= 2\cos 4x + \sqrt{2}(\sin x - \cos x)$

$\sqrt{2}\sin(x - \frac{\pi}{4})$

$= 2\cos 4x + 2\sin(x - \frac{\pi}{4}) \xrightarrow{x = \frac{\pi}{12}}$

$= 2\cos\frac{\pi}{3} + 2\sin(\frac{\pi}{12} - \frac{\pi}{4}) = 2\times\frac{1}{2} + 2\times(\frac{-1}{2}) = \frac{1}{2}$

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = ?$ $\quad \tan\frac{\alpha}{2} = \frac{1}{4}$ ①

$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{2(\frac{1}{4})}{1+\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{17}{16}} = \frac{8}{17}$

$\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{1-\frac{1}{16}}{1+\frac{1}{16}} = \frac{\frac{15}{16}}{\frac{17}{16}} = \frac{15}{17}$

$\tan(\alpha) = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{8}{17}}{\frac{15}{17}} = \frac{8}{15}$

$\rightarrow = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{-17}{105}$

⑩

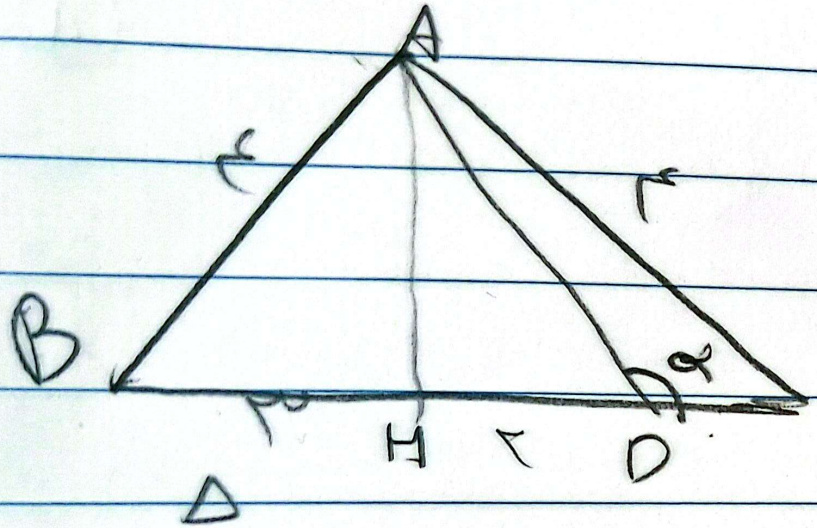
$2\sin\alpha < \sin^2\alpha \rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha$

$\rightarrow 2\sin\alpha - 2\sin\alpha\cos\alpha < 0$

$\rightarrow 2\sin\alpha(\underbrace{1-\cos\alpha}_{\text{همواره مثبت}}) < 0 \Rightarrow \sin\alpha < 0$ (☆)

$\frac{\cos^2\alpha}{\sin\alpha} > 0 \rightarrow \frac{\cos\alpha}{\sin^2\alpha} > 0 \rightarrow \cos\alpha > 0$ (✱)

(☆) و (✱) $\Rightarrow$ ربع چهارم



(۴)

$AM^2 + BM^2 = AB^2$

$16 = (AM)^2 + 9 \Rightarrow (AM)^2 = 7$

$\Rightarrow AM = \sqrt{7}$

AMD: $\tan(\pi - \alpha) = \frac{\sqrt{7}}{3} \Rightarrow -\tan\alpha = -\frac{\sqrt{7}}{3} \Rightarrow \tan\alpha = \frac{\sqrt{7}}{3}$

V) $\sin\left(\frac{9\pi}{2}+\alpha\right)=\cos\alpha$ $\qquad$ $\cos\left(\frac{5\pi}{2}-\alpha\right)=-\sin\alpha$

$\tan\left(\alpha-\frac{3\pi}{2}\right)=-\cot\alpha$ $\qquad$ $\rightarrow \frac{-\mu}{\omega}\times\frac{\kappa}{\lambda}+\frac{\mu}{\kappa}=\frac{\mu\nu}{\iota\omega}$