آزمونک یازدهم ریاضی B تکلیف ۲۷

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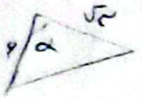
(۱)

$S=\frac{1}{2}\times 6\times\sqrt{3}\times\sin\alpha=\frac{9}{2}$



$\sin\alpha=\frac{\sqrt{3}}{2}\Rightarrow$ یا $60^\circ \pm \alpha$ ، $120^\circ \pm \alpha$

$\boxed{\frac{120}{60}=2}$

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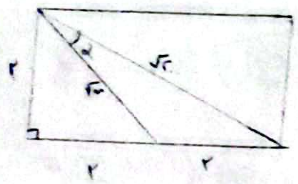
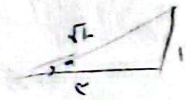


$\cot\alpha=?$

$S_{\text{مثلث}}=4\times 2-\left(\frac{4}{2}+\frac{8}{2}\right)=2$

$S_{\text{مثلث}}=\sqrt{20}\times\frac{1}{2}\times\sqrt{8}\times\sin\alpha=2\longrightarrow\sin\alpha=\frac{1}{\sqrt{10}}$



$\longrightarrow \boxed{\cot\alpha=\frac{3}{1}}$

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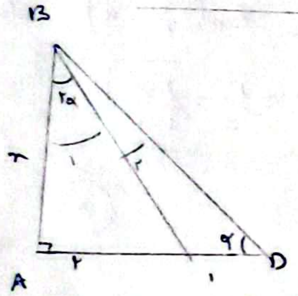
(۳)



$\cot\alpha=?\quad \tan B_1=\frac{AC}{AB}\rightarrow\tan 2\alpha=\frac{2}{x}$

$\tan D=\frac{AB}{AD}\rightarrow\tan\alpha=\frac{x}{3}$

$\tan 2\alpha=\frac{\frac{2x}{3}}{\frac{9-x^2}{9}}\rightarrow\frac{2}{x}=\frac{6x}{9-x^2}\longrightarrow 18-2x^2=6x^2\longrightarrow 8x^2=18$

$\cot\alpha=\frac{AD}{AB}=\frac{3}{\cancel{x}}\Rightarrow\frac{3}{\frac{3}{2}}\ 2$

$\boxed{x^2=\frac{9}{4}\longrightarrow x=\frac{3}{2}}$

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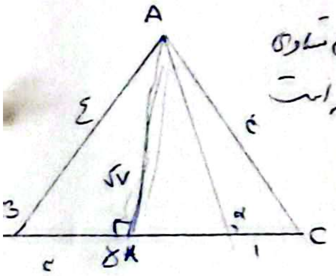

چون متساوی الساقین $AB=AC=4$

متساوی الساقین $\Leftarrow$ AH میانه است

$\tan(\pi-\alpha)=-\tan\alpha=\frac{\sqrt{7}}{3}\longrightarrow\boxed{\tan\alpha=\frac{-\sqrt{7}}{3}}$

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(۵)

$2\sin^2x+\cos^2x=\frac{4}{3}\qquad \tan^2x=?$

$\div\cos^2\alpha\quad 2\tan^2x+1=\frac{4}{3}\left(1+\tan^2x\right)$

$6\tan^2x+3=4+4\tan^2x\longrightarrow 2\tan^2x=1\longrightarrow\boxed{\tan^2x=\frac{1}{2}}$

Scanned with CamScanner

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$$A = \frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha}$$

$$A = \frac{(1-\cos^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(1-\sin^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha}$$

$$A = \frac{(1+\cos^2\alpha)^2}{1+\cos^2\alpha} - \frac{(1+\sin^2\alpha)^2}{1+\sin^2\alpha} = \boxed{\cos 2\alpha}$$

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$$\tan\alpha = \frac{4}{3} \qquad \underbrace{\sin\left(\frac{9\pi}{2}+\alpha\right)}_{-\frac{3}{5}}\underbrace{\cos\left(\frac{7\pi}{2}-\alpha\right)}_{\frac{4}{5}} - \overbrace{\tan\left(\alpha-\frac{3\pi}{2}\right)}^{\frac{3}{4}} = ?$$

$$1+\frac{16}{9} = \frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha = \frac{9}{25} \xrightarrow{\text{ربع سوم}} \cos\alpha = -\frac{3}{5}$$

$$\sin^2\alpha = 1-\cos^2\alpha \rightarrow \sin^2\alpha = \frac{16}{25} \xrightarrow{\text{ربع سوم}} \sin\alpha = -\frac{4}{5}$$

$$\cot\alpha = \frac{3}{4}$$

$$\sin\left(\frac{9\pi}{2}+\alpha\right) = \left(\sin\left(4\pi+\frac{\pi}{2}+\alpha\right) = \sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha = -\frac{3}{5}$$

$$\cos\left(\frac{7\pi}{2}-\alpha\right) = \cos\left(2\pi+\frac{3\pi}{2}-\alpha\right) = \cos\left(\frac{3\pi}{2}-\alpha\right) \rightarrow \sin\alpha = \frac{4}{5}$$

$$\tan\left(\alpha-\frac{3\pi}{2}\right) = -\tan\left(\frac{3\pi}{2}-\alpha\right) = -\cot\alpha = -\frac{3}{4}$$

$$-\frac{3}{5}\times\frac{4}{5}+\frac{3}{4} = \boxed{\frac{27}{100}}$$

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$$(2\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x)$$

$$x = \frac{\pi}{12} \qquad \cos\frac{4\pi}{12} = \frac{1}{2}$$

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$$\sqrt{2}(\sin x - \cos x) = \sqrt{2}\times\sqrt{2}\sin\left(x-\frac{\pi}{4}\right) = 2\sin\left(-\frac{\pi}{6}\right) = -1$$

$$\text{جواب} \rightarrow \frac{3}{2} - 1 = \boxed{\frac{1}{2}}$$

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$$\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}} = \boxed{-\frac{16}{105}}$$

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = ?$$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{4} \qquad \tan\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{8}{15}$$

$$1+\tan^2\alpha = \frac{1}{\cos^2\alpha}$$

$$1+\frac{64}{225} = \frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha = \frac{225}{289}$$

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$$\cos\alpha = \frac{15}{17} \rightarrow \sin\alpha = \sqrt{1-\frac{225}{289}} = \boxed{\frac{8}{17}}$$

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(۱۰) ۲

$$2\sin\alpha < \sin 2\alpha \qquad \frac{\cot\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sin^2\alpha}$$

چون $\sin^2\alpha$ همیشه مثبت است علامت با $\cos\alpha$

$$\frac{\cot\alpha}{\sin\alpha} > 0$$ است پس $\cos\alpha$

$\cos\alpha > 0 \Rightarrow$ در ربع اول یا چهارم

ربع ۱ یا ۴ : $\cos\alpha > 0$

ربع ۳ یا ۴ : $\sin\alpha < 0$

تقاطع فقط ربع ۴

$$0 < \frac{\cot\alpha}{\sin\alpha}$$

ربع ۴ ← جواب

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$2\sin\alpha\cos\alpha > 2\sin\alpha$$

$$\sin\alpha(\cos\alpha - 1) > 0$$ داریم فاکتور میگیریم

چون $\cos\alpha - 1$ فقط در صورتی صفر میشه که $\cos\alpha = 1$ برای اینکه حاصل ضرب بزرگتر از صفر باشد باید $\sin\alpha < 0$ چون $(\cos\alpha - 1) < 0$ منفی است