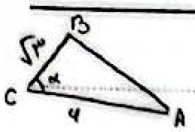
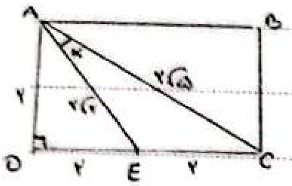
Subject:
Date

یلدا ادریسی ۱۵ — تکلیف شماره ۲۷



$S=\frac{1}{2}ab\sin\alpha \Rightarrow \frac{1}{2}\times\sqrt{3}\times 4\times\sin\alpha=3 \Rightarrow \sin\alpha=\frac{\sqrt{3}}{2} \begin{cases}\alpha=60^\circ\\ \alpha=120^\circ\end{cases} \Rightarrow \frac{120}{60}=\underline{2}$ ۱-

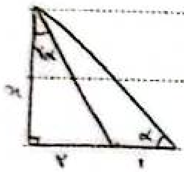


$AE=\sqrt{4+4}=2\sqrt{2}$ $\qquad AC=\sqrt{4+16}=2\sqrt{5}$ ۲-

$EC^2=AC^2+AE^2-2AC\cdot AE\cos\alpha \Rightarrow 4=8+20-8\sqrt{10}\cos\alpha$

$\Rightarrow 8\sqrt{10}\cos\alpha=24 \Rightarrow \cos\alpha=\frac{3}{\sqrt{10}} \rightsquigarrow \sin^2\alpha=1-\frac{9}{10} \Rightarrow \sin\alpha=\frac{1}{\sqrt{10}}$

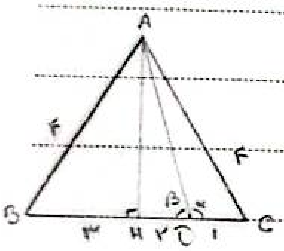
$\Rightarrow \cot\alpha=\dfrac{\frac{3}{\sqrt{10}}}{\frac{1}{\sqrt{10}}}=\underline{3}$



$\cot\alpha=\frac{3}{x}$, $\cot 2\alpha=\frac{x}{2}$ $\qquad \cot 2\alpha=\frac{\cot^2\alpha-1}{2\cot\alpha}$ ۳-

$\Rightarrow \frac{x}{2}=\dfrac{\frac{9-x^2}{x^2}}{\frac{6}{\cancel{x}}}=\frac{9-x^2}{6x} \Rightarrow 6x^2=18-2x^2 \Rightarrow x^2=\frac{9}{4} \to x=\frac{3}{2}$

$\cot\alpha=\dfrac{3}{\frac{3}{2}}=\underline{2}$



$AH=\sqrt{16-9}=\sqrt{7}$ $\qquad \tan(\underbrace{\pi-\alpha}_{\beta})=-\tan\alpha=\frac{\sqrt{7}}{2}$ ۴-

$\Rightarrow \tan\alpha=\underline{\frac{-\sqrt{7}}{2}}$

$2\sin^2x+\cos^2x=\frac{4}{3} \Rightarrow \sin^2x+1=\frac{4}{3} \Rightarrow \sin^2x=\frac{1}{3}$, $\cos^2x=1-\frac{1}{3}=\frac{2}{3}$ ۵-

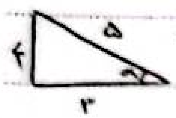
$1+\tan^2x=\frac{1}{\cos^2x} \Rightarrow \tan^2x=\frac{3}{2}-1=\underline{\frac{1}{2}}$

$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{\cos^4\alpha+2\cos^2\alpha+1}{1+\cos^2\alpha}-\frac{\sin^4\alpha+2\sin^2\alpha+1}{1+\sin^2}=\frac{(1+\cos^2\alpha)^2}{1+\cos^2\alpha}-\frac{(\sin^2\alpha+1)^2}{1+\sin^2\alpha}$ ۶-

$=\cancel{1}+\cos^2\alpha-\sin^2\alpha-\cancel{1}=\underline{\cos 2\alpha}$

-۷ $\sin(\frac{9\pi}{2}+\alpha)\cos(\frac{7\pi}{2}-\alpha)-\tan(\alpha-\frac{3\pi}{2}) = \sin(\frac{\pi}{2}+\alpha)\cos(\frac{3\pi}{2}-\alpha)+\tan(\frac{3\pi}{2}-\alpha)$

$= \cos\alpha(-\sin\alpha)+\cot\alpha$



$\rightarrow \sin\alpha = -\frac{4}{5}$, $\cos\alpha = -\frac{3}{5}$

$-\frac{3}{5}\times\frac{4}{5}+\frac{3}{4} = 0/27$

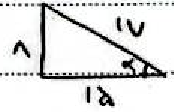
-۸ $\sin x-\cos x = \sqrt{2}\sin(x-\frac{\pi}{4})$

$\underbrace{\sqrt{2}\sin(15-45)}$

$3\cos 4x+\sqrt{2}\sin x-\sqrt{2}\cos x = 3\cos 60+\sqrt{2}(\sin 15-\cos 15) = 3\cos 60+2\sin(-30) = 3\cos 60-2\sin 30$

$= 3\times\frac{1}{2}-2\times\frac{1}{2} = \frac{1}{2}$

۹ $\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{8}{15}$



$\sin\alpha = \frac{8}{17}$ $\quad \cos\alpha = \frac{15}{17}$

$\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}} = \frac{-16}{105}$

-۱۰ $\frac{\cot x}{\sin x} > 0 \Rightarrow \frac{\cos x}{\sin^2 x} > 0 \Rightarrow \cos x > 0 \rightarrow$ اول یا چهارم

نیست، ولی تساوی

$2\sin\alpha < \sin 2\alpha \rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \Rightarrow \sin\alpha\cos\alpha-\sin\alpha > 0 \rightarrow \sin\alpha(\underbrace{\cos\alpha-1}_{-}) > 0 \Rightarrow \sin\alpha < 0$

ناحیه چهارم