(تکلیف ۲۷)

(فاطمه شکرانی)

۲۰

(یازدهم ریاضی B)

①

$S_{\text{مثلث}} = \frac{1}{2} a b \sin\alpha \to 4/5 = \frac{1}{2}\times 6 \times \sqrt{3} \times \sin\alpha \to \sin\alpha = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \to \alpha \begin{cases} \to 120^\circ \\ \to 60^\circ \end{cases}$

$\frac{\alpha \ max}{\alpha \ min} = \frac{120}{60} = \boxed{2}$

✓

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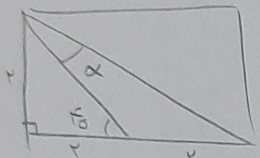
②



$\cot(\alpha + 45) = \frac{2}{4} = \frac{1}{2}$

$\cot(\alpha + 45) = \frac{\cot\alpha \cot 45 - 1}{\cot\alpha + \cot 45} = \frac{\cot\alpha \times 1 - 1}{\cot\alpha + 1} = \frac{1}{2}$

$\to 2\cot\alpha - 2 = \cot\alpha + 1$

$\boxed{\cot\alpha = 3}$

✓

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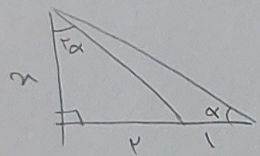
③



$\left.\begin{aligned} \tan 2\alpha &= \frac{2}{x} \\ \tan\alpha &= \frac{x}{3} \\ \tan 2\alpha &= \frac{2\tan\alpha}{1-\tan^2\alpha} \end{aligned}\right\} \to \frac{2\left(\frac{x}{3}\right)}{1-\left(\frac{x^2}{9}\right)} = \frac{2}{x} \to \frac{\frac{2x}{3}}{\frac{9-x^2}{9}} = \frac{2}{x} \to \frac{6x}{9-x^2} = \frac{2}{x}$

$6x^2 = 18 - 2x^2 \to 8x^2 = 18$

$x^2 = \frac{9}{4}$

$x = \frac{3}{2}$

چون x مقدار ضلع است نمی‌تواند منفی باشد

$\cot\alpha = \frac{3}{x} = \frac{3}{\frac{3}{2}} = \boxed{2}$

✓

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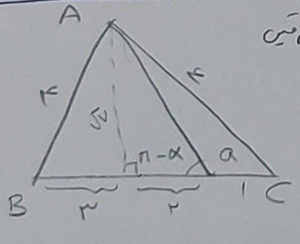
④



میانه همان ارتفاع است → مثلث متساوی الساقین

$AH = \sqrt{16-9} = \sqrt{7}$

$\tan(\pi - \alpha) = \frac{\sqrt{7}}{2}$

$-\tan\alpha = \frac{\sqrt{7}}{2}$

$\boxed{\tan\alpha = \frac{-\sqrt{7}}{2}}$

✓

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⑤

$\underbrace{2\sin^2 x + \cos^2 x}_{1 + \sin^2 x} = \frac{4}{3}$

$1 + \sin^2 x = \frac{4}{3} \to \sin^2 x = \frac{1}{3} \to \sin x = \begin{cases} +\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{cases}$

$\cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{3} = \frac{2}{3}$

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{\frac{1}{2}}$

✓

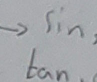
$$\frac{\sin^{۴}\alpha+۴\cos^{۲}\alpha}{۱+\cos^{۲}\alpha}-\frac{\cos^{۴}\alpha+۴\sin^{۲}\alpha}{۱+\sin^{۲}\alpha}=\frac{\sin^{۴}\alpha+۴(۱-\sin^{۲}\alpha)}{۲-\sin^{۲}\alpha}-\frac{\cos^{۴}\alpha+۴(۱-\cos^{۲}\alpha)}{۲-\cos^{۲}\alpha}$$

$$=\frac{\sin^{۴}\alpha-۴\sin\alpha+۴}{۲-\sin^{۲}\alpha}-\frac{\cos^{۴}\alpha-۴\cos^{۲}\alpha+۴}{۲-\cos^{۲}\alpha}=\frac{(\sin^{۲}\alpha-۲)^{۲}}{۲-\sin^{۲}\alpha}-\frac{(\cos^{۲}\alpha-۲)^{۲}}{۲-\cos^{۲}\alpha}$$ ⑥

$$=(۲-\sin^{۲}\alpha)-(۲-\cos^{۲}\alpha)=\cos^{۲}\alpha-\sin^{۲}\alpha=\boxed{\cos ۲\alpha}$$

$$\sin\left(\frac{۹\pi}{۲}+\alpha\right)\cos\left(\frac{۷\pi}{۲}-\alpha\right)-\underbrace{\tan\left(\alpha-\frac{۳\pi}{۲}\right)}_{-\tan\left(\frac{۳\pi}{۲}-\alpha\right)}=?$$

$\tan\alpha=\frac{۴}{۳}$, α در ربع ۳



$\rightarrow \sin, \cos < ۰$

$\tan, \cot > ۰$

$$=\cos\alpha\times(-\sin\alpha)+\cot\alpha=\frac{-۳}{۵}\times\frac{۴}{۵}+\frac{۳}{۴}=\frac{-۱۲}{۲۵}+\frac{۳}{۴}=\frac{-۴۸+۷۵}{۱۰۰}=\boxed{\frac{۲۷}{۱۰۰}}$$ ⑦

$$۳\cos ۴x+\sqrt{۲}\sin x-\sqrt{۲}\cos x \quad , x=\frac{\pi}{۱۲}$$

$$=۳\cos ۴x+\sqrt{۲}\underbrace{(\sin x-\cos x)}_{\sqrt{۲}\sin\left(x-\frac{\pi}{۴}\right)}=۳\cos ۴\left(\frac{\pi}{۱۲}\right)+۲\sin\left(\frac{\pi}{۱۲}-\frac{\pi}{۴}\right)=۳\cos\frac{\pi}{۳}+۲\sin\left(\frac{-\pi}{۶}\right)$$

$$=\frac{۳}{۲}-۱=\boxed{\frac{۱}{۲}}$$ ⑧

$$\tan\alpha=\tan ۲\left(\frac{\alpha}{۲}\right)=\frac{۲\tan\left(\frac{\alpha}{۲}\right)}{۱-\tan^{۲}\left(\frac{\alpha}{۲}\right)}=\frac{\frac{۱}{۲}}{\frac{۱۵}{۱۶}}=\frac{۱۶}{۳۰}=\frac{۸}{۱۵}$$

$$\cos\alpha=\cos ۲\left(\frac{\alpha}{۲}\right)=\frac{۱-\tan^{۲}\left(\frac{\alpha}{۲}\right)}{۱+\tan^{۲}\left(\frac{\alpha}{۲}\right)}=\frac{\frac{۱۵}{۱۶}}{\frac{۱۷}{۱۶}}=\frac{۱۵}{۱۷}$$

$$\sin\alpha=\sin ۲\left(\frac{\alpha}{۲}\right)=\frac{۲\tan\left(\frac{\alpha}{۲}\right)}{۱+\tan^{۲}\left(\frac{\alpha}{۲}\right)}=\frac{\frac{۱}{۲}}{\frac{۱۷}{۱۶}}=\frac{۸}{۱۷}$$

$$\longrightarrow \frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{۸}{۱۵}-\frac{۸}{۱۷}}{\frac{۸}{۱۷}-\frac{۱۵}{۱۷}}=\boxed{\frac{-۱۶}{۱۰۵}}$$ ⑨

$$۲\sin\alpha<\sin ۲\alpha \quad , \quad ۰<\frac{\cot\alpha}{\sin\alpha}$$

α در چه ربعی؟

$$\frac{\cot\alpha}{\sin\alpha}>۰\longrightarrow\frac{\cos\alpha}{\underbrace{\sin^{۲}\alpha}_{\text{همواره مثبت}}}>۰\longrightarrow\cos\alpha>۰ \quad (I)$$

(I) , (II) $\rightarrow$ ربع ۴

⑩

$$۲\sin\alpha<\sin ۲\alpha\longrightarrow \cancel{۲}\sin\alpha<\cancel{۲}\sin\alpha\cos\alpha\longrightarrow\sin\alpha\cos\alpha-\sin\alpha>۰$$

$$\sin\alpha\underbrace{(\cos\alpha-۱)}_{\ominus}>۰\longrightarrow\sin\alpha<۰ \quad (II)$$