(تکلیف ۲۷)

(فاطمه تهرانی)

(یازدهم ریاضی B)

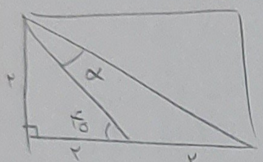
۱)

$$S_{\text{مثلث}} = \frac{1}{2} a b \sin\alpha \to 4{,}5 = \frac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha \to \sin\alpha = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \to \alpha \begin{cases} 120^\circ \\ 60^\circ \end{cases}$$

$$\frac{\alpha\ max}{\alpha\ min} = \frac{120}{60} = 2$$

---

۲)



$$\cot(\alpha + 45) = \frac{2}{4} = \frac{1}{2}$$

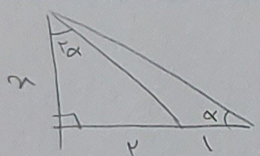
$$\cot(\alpha + 45) = \frac{\cot\alpha \cot 45 - 1}{\cot\alpha + \cot 45} = \frac{\cot\alpha \times 1 - 1}{\cot\alpha + 1} = \frac{1}{2}$$

$$\to 2\cot\alpha - 2 = \cot\alpha + 1$$

$$\boxed{\cot\alpha = 3}$$

---

۳)



$$\left.\begin{aligned} \tan 2\alpha &= \frac{2}{x} \\ \tan\alpha &= \frac{x}{3} \\ \tan 2\alpha &= \frac{2\tan\alpha}{1 - \tan^2\alpha} \end{aligned}\right\} \to \frac{2\left(\frac{x}{3}\right)}{1 - \left(\frac{x^2}{9}\right)} = \frac{2}{x} \to \frac{\frac{2x}{3}}{\frac{9 - x^2}{9}} = \frac{2}{x} \to \frac{6x}{9 - x^2} = \frac{2}{x}$$

$$6x^2 = 18 - 2x^2 \to 8x^2 = 18$$

$$x^2 = \frac{9}{4}$$

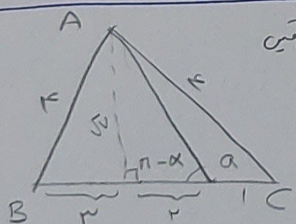
$$x = \frac{3}{2}$$

چون x مقدار ضلع است نمی‌تواند منفی باشد

$$\cot\alpha = \frac{3}{x} = \frac{3}{\frac{3}{2}} = \boxed{2}$$

---

۴)



میانه همان ارتفاع است ← مثلث متساوی الساقین

$$AH = \sqrt{16 - 9} = \sqrt{7}$$

$$\tan(\pi - \alpha) = \frac{\sqrt{7}}{2}$$

$$-\tan\alpha = \frac{\sqrt{7}}{2}$$

$$\boxed{\tan\alpha = \frac{-\sqrt{7}}{2}}$$

---

۵)

$$\underbrace{2\sin^2 x + \cos^2 x}_{1 + \sin^2 x} = \frac{4}{3}$$

$$1 + \sin^2 x = \frac{4}{3} \to \sin^2 x = \frac{1}{3} \to \sin x = \begin{cases} +\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{cases}$$

$$\cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{\frac{1}{2}}$$

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{2-\sin^2\alpha} - \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{2-\cos^2\alpha}$$

$$= \frac{\sin^4\alpha - 4\sin^2\alpha + 4}{2-\sin^2\alpha} - \frac{\cos^4\alpha - 4\cos^2\alpha + 4}{2-\cos^2\alpha} = \frac{(\sin^2\alpha - 2)^2}{2-\sin^2\alpha} - \frac{(\cos^2\alpha - 2)^2}{2-\cos^2\alpha}$$ ۶

$$= (2-\sin^2\alpha) - (2-\cos^2\alpha) = \cos^2\alpha - \sin^2\alpha = \boxed{\cos 2\alpha}$$

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \underbrace{\tan\left(\alpha - \frac{3\pi}{2}\right)}_{-\tan\left(\frac{3\pi}{2}-\alpha\right)} = ? \qquad \tan\alpha = \frac{4}{3}\ ,\ \alpha \text{ در ربع } 3$$

$\to$ 3, 4, 5 $\qquad \to \sin, \cos < 0$; $\tan, \cot > 0$

$$= \cos\alpha \times (-\sin\alpha) + \cot\alpha = \frac{-3}{5}\times\frac{4}{5} + \frac{3}{4} = \frac{-12}{25} + \frac{3}{4} = \frac{-48+75}{100} = \boxed{\frac{27}{100}}$$ ۷

$$3\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x \quad , \quad x = \frac{\pi}{12}$$

$$= 3\cos 4x + \sqrt{2}(\underbrace{\sin x - \cos x}_{\sqrt{2}\sin(x-\frac{\pi}{4})}) = 3\cos 4\left(\frac{\pi}{12}\right) + 2\sin\left(\frac{\pi}{12}-\frac{\pi}{4}\right) = 3\cos\frac{\pi}{3} + 2\sin\left(\frac{-\pi}{6}\right)$$

$$= \frac{3}{2} - 1 = \boxed{\frac{1}{2}}$$ ۸

$$\tan\alpha = \tan 2\left(\frac{\alpha}{2}\right) = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{16}{30} = \frac{8}{15}$$

$$\cos\alpha = \cos 2\left(\frac{\alpha}{2}\right) = \frac{1-\tan^2\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{\frac{15}{16}}{\frac{17}{16}} = \frac{15}{17}$$ ۹

$$\sin\alpha = \sin 2\left(\frac{\alpha}{2}\right) = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{\frac{1}{2}}{\frac{17}{16}} = \frac{8}{17} \quad \longrightarrow \quad \frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \boxed{\frac{-16}{105}}$$

$$2\sin\alpha < \sin 2\alpha \quad , \quad 0 < \frac{\cot\alpha}{\sin\alpha} \qquad \alpha \text{ در چه ربعی؟}$$

$$\frac{\cot\alpha}{\sin\alpha} > 0 \to \frac{\cos\alpha}{\underbrace{\sin^2\alpha}_{\text{همواره مثبت}}} > 0 \to \cos\alpha > 0 \quad \text{I} \qquad\qquad \text{I}, \text{II} \to \text{ربع } 4$$ ۱۰

$$2\sin\alpha < \sin 2\alpha \to 2\sin\alpha < 2\sin\alpha\cos\alpha \to \sin\alpha\cos\alpha - \sin\alpha > 0$$

$$\sin\alpha(\underbrace{\cos\alpha - 1}_{-}) > 0 \to \sin\alpha < 0 \quad \text{II}$$