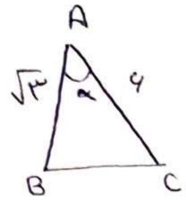
شادان کشاورزیان — شماره‌ی تکلیف: ۲۷ — مقطع: یازدهم دختر B

۱۸٫۵

۱



$S_{\overset{\triangle}{ABC}} = \frac{1}{۲} AB \times AC \times \sin\alpha = ۴٫۵$

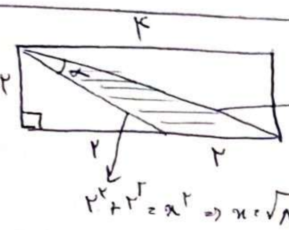
$۹ = ۴\sqrt{۳} \times \sin\alpha \Rightarrow \frac{۳}{۲\sqrt{۳}} \times \frac{\sqrt{۳}}{\sqrt{۳}} = \frac{۳\sqrt{۳}}{۶} = \frac{\sqrt{۳}}{۲} = \sin\alpha$

$\alpha = ۶۰$ کمترین مقدار

$\alpha = ۱۲۰$ بیشترین مقدار

$\frac{۱۲۰}{۶۰} = ۲$ برابر

۲



$۲^۲ + ۴^۲ = (y)^۲ \Rightarrow y = \sqrt{۲۰}$

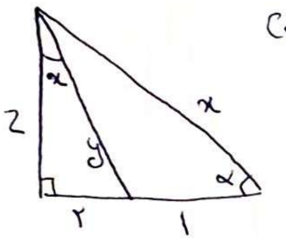
$۲^۲ + ۲^۲ = x^۲ \Rightarrow x = \sqrt{۸}$

S مثلث هاشورخورده $= ۸ - ۲ - ۴ = ۲ = \frac{1}{۲} \times ۲\sqrt{۲} \times ۲\sqrt{۵} \sin\alpha \Rightarrow \sin\alpha = \frac{1}{\sqrt{۱۰}}$

$(\sqrt{۱۰})^۲ = ۱^۲ + z^۲ \Rightarrow z = ۳$

$\Rightarrow \boxed{\cot\alpha = ۳}$

۳



$\cos ۲\alpha = \cos^۲\alpha - \sin^۲\alpha$

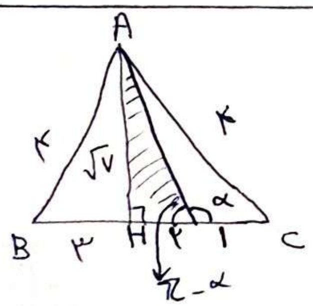
$\sin ۲\alpha = ۲\sin\alpha\cos\alpha$

$\Rightarrow \begin{cases} \frac{z}{y} = \left(\frac{۳}{x}\right)^۲ - \left(\frac{z}{x}\right)^۲ \Rightarrow \frac{۹ - z^۲}{x^۲} = \frac{z}{y} \\ \frac{۳}{y} = ۲ \times \frac{z}{x} \times \frac{۳}{x} \Rightarrow x^۲ = ۲yz \end{cases}$

$\frac{۹ - z^۲}{۲yz} = \frac{z}{y} \Rightarrow z = \frac{۳}{۲}$

$\Rightarrow \cot\alpha = \frac{۳}{z} = ۲$ (جایگزینی)

۴



$AB^۲ = AH^۲ + BH^۲ \Rightarrow ۱۶ = AH^۲ + ۹ \Rightarrow AH^۲ = ۷ \Rightarrow AH = \sqrt{۷}$

$\tan(\pi - \alpha) = \frac{\sqrt{۷}}{۲} \Rightarrow -\tan\alpha = \frac{\sqrt{۷}}{۲}$

$\Rightarrow \tan\alpha = -\frac{\sqrt{۷}}{۲}$

۵

$۲\sin^۲ x + \cos^۲ x = \frac{۴}{۳}$

$\rightarrow \sin^۲ x + \cos^۲ x = ۱ \Rightarrow ۱ - \sin^۲ x = \cos^۲ x$

$\Rightarrow \sin^۲ x = \frac{۱}{۳}$ ، همچنین $\cos^۲ x = ۱ - \frac{۱}{۳} = \frac{۲}{۳}$

$\tan^۲ x = \frac{\sin^۲ x}{\cos^۲ x} = \frac{۱}{۳} \times \frac{۳}{۲} = \boxed{\frac{۱}{۲}}$

$$\frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{2-\sin^2\alpha} - \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{2-\cos^2\alpha} \Rightarrow \frac{\sin^4\alpha - 4\sin^2\alpha + 4}{2-\sin^2\alpha} - \frac{\cos^4\alpha - 4\cos^2\alpha + 4}{2-\cos^2\alpha} =$$

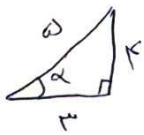
$$\Rightarrow \frac{(\sin^2\alpha - 2)^2}{2-\sin^2\alpha} - \frac{(\cos^2\alpha - 2)^2}{2-\cos^2\alpha} = (2-\sin^2\alpha) - (2-\cos^2\alpha) = \cos^2\alpha$$

1,75 — 4

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right) = \sin\left(\frac{\pi}{2}+\alpha\right)\cos\left(\frac{3\pi}{2}-\alpha\right) + \tan\left(\frac{3\pi}{2}-\alpha\right)$$

اگر α $\Rightarrow \cos(\alpha)(-\sin(\alpha)) + \cot(\alpha)$

$\rightarrow \tan\alpha = \frac{4}{3}$ α سوم $\Rightarrow \tan\alpha = \frac{3}{3}$



$$\Rightarrow \begin{cases} \cos\alpha = -\frac{3}{5} \\ \sin\alpha = -\frac{4}{5} \\ \cot\alpha = \frac{3}{4} \end{cases}$$

$$\left(-\frac{3}{5}\right)\left(-\left(-\frac{4}{5}\right)\right) + \frac{3}{4} = -\frac{12}{25} + \frac{3}{4} = -\frac{48+75}{100} = \frac{27}{100}$$

6 — 7

$$\frac{3\cos 4x + \sqrt{2}(\sin x - \cos x)}{\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)}$$

$$\Rightarrow \frac{3\cos 4x + 2\sin\left(x - \frac{\pi}{4}\right)}{}$$

$x = \frac{\pi}{12} \rightarrow 3\cos\left(\frac{\pi}{3}\right) + 2\sin\left(\underbrace{\frac{\pi}{12} - \frac{\pi}{4}}_{-\frac{\pi}{6}}\right) = 3\times\frac{1}{2} + 2\times\frac{1}{2} = \frac{1}{2}$

6 — 8

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{4}$$

$$\sin\alpha = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{\frac{2}{4}}{1+\frac{1}{16}} = \frac{8}{17}$$

$$\cos\alpha = \frac{1-\tan^2\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{1-\frac{1}{16}}{1+\frac{1}{16}} = \frac{\frac{15}{16}}{\frac{17}{16}} = \frac{15}{17}$$ (5)

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{\frac{8}{17}}{\frac{15}{17}} = \frac{8}{15} \Rightarrow \frac{\frac{8}{15} - \frac{7}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{2}{15\times 17}}{\frac{-7}{17}} = -\frac{8\times 2}{15\times 7} = -\frac{16}{105}$$

4

0,75

$$2\sin\alpha < \sin 2\alpha$$

$$0 < \frac{\cot\alpha}{\sin\alpha} \Rightarrow 0 < \frac{\cos\alpha}{\sin^2\alpha} \Rightarrow \cos\alpha > 0 \Rightarrow$$ ربع اول یا چهارم

اگر ربع اول $\Rightarrow$ یک مثال $\alpha = 30^\circ$ $\Rightarrow 2\sin\frac{1}{2} < \sin(60^\circ) \Rightarrow \frac{\sqrt{3}}{2} > 1$ نقض شد

اگر ربع چهارم $\Rightarrow \alpha = 300^\circ \Rightarrow 2\sin(-30^\circ) < -\sin 60^\circ \rightarrow -1 < -\frac{\sqrt{3}}{2}$ صدق کرد

پس انتهای کمان ربع اول بود

10

$$\begin{cases} \frac{\cot\alpha}{\sin\alpha} > 0 \rightarrow \frac{\cos}{\sin^2\alpha} > 0 \\ 2\sin\alpha < \sin 2\alpha \rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \end{cases} \rightarrow \begin{cases} \sin^2\alpha > 0 \rightarrow \cos\alpha > 0 \\ \sin\alpha(\cos\alpha - \sin\alpha) > 0 \rightarrow \sin\alpha < 0 \end{cases}$$

$\rightarrow$ ناصحیح