نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۷ ... کلاس یازدهم ریاضی .....

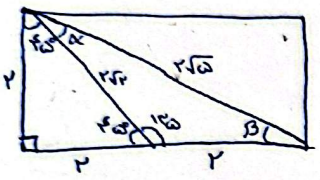
(۲۰)

۱

$S_{ABC} = 4\sqrt{3} \Rightarrow \frac{\sqrt{3} \times 4 \times \sin\alpha}{2} = \frac{3}{2} \Rightarrow \sin\alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha_1 = 60^\circ$

$\alpha_2 = 120^\circ \Rightarrow \frac{\alpha_2}{\alpha_1} = \boxed{2}$

۲



$\alpha + \beta = 180 - 135 = 45$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$\Rightarrow 2\sin\alpha + \cos\alpha = \frac{\sqrt{2}}{2} \times \sqrt{5}$ (۱)

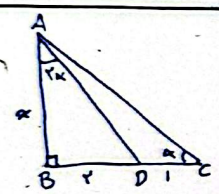
$\sin 45^\circ = \sin\alpha \times \frac{2}{2\sqrt{5}} + \frac{2}{2\sqrt{5}}\cos\alpha = \frac{2\sin\alpha + \cos\alpha}{\sqrt{5}}$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \Rightarrow \cos 45^\circ = \cos\alpha \times \frac{2}{2\sqrt{5}} - \frac{2}{2\sqrt{5}}\sin\alpha = \frac{2\cos\alpha - \sin\alpha}{\sqrt{5}}$

$\Rightarrow 2\cos\alpha - \sin\alpha = \frac{\sqrt{2}}{2} \times \sqrt{5}$ (۲)

(۱), (۲) $\Rightarrow 2\cos\alpha - \sin\alpha = 2\sin\alpha + \cos\alpha \Rightarrow \cos\alpha = 3\sin\alpha \Rightarrow \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \boxed{3}$

۳



$AD = \sqrt{4+x^2}$

$AC = \sqrt{9+x^2}$

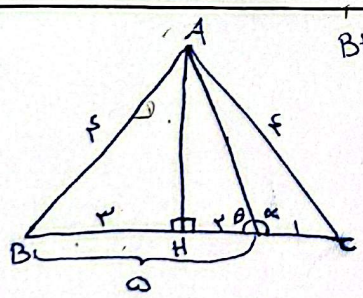
$\cot 2\alpha = \frac{x}{2} = \frac{\cos 2\alpha}{\sin 2\alpha} = \frac{\cos^2\alpha - \sin^2\alpha}{2\sin\alpha\cos\alpha}$

$\Rightarrow \frac{x}{2} = \frac{\frac{4}{x^2+4} - \frac{x^2}{x^2+9}}{2 \times \frac{x}{\sqrt{x^2+4}} \times \frac{3}{\sqrt{x^2+9}}} = \frac{\frac{9-x^2}{(x^2+4)(x^2+9)}}{\frac{6x}{(x^2+4)(x^2+9)}} = \frac{9-x^2}{6x}$

$\Rightarrow 18 - 2x^2 = 6x^2 \Rightarrow 8x^2 = 18 \Rightarrow x^2 = \frac{18}{8} = \frac{9}{4} \Rightarrow x = \frac{3}{2}$

$\cot\alpha = \frac{3}{x} = \frac{3}{\frac{3}{2}} = \boxed{2}$

۴



در مثلث متساوی الساقین ارتفاع وارد بر قاعده ($\overline{BC}$) عمود منصف آن است. $\Leftarrow BH = CH = 3$

$AH = \sqrt{5^2 - 3^2} = \sqrt{7}$

$\tan\alpha = -\tan\theta = \boxed{-\frac{\sqrt{7}}{2}}$

۵

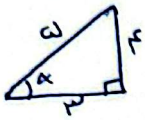
$\sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \frac{4}{3} \Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \sin x = \pm\frac{1}{\sqrt{3}}$

$\tan^2\alpha = \left(\pm\frac{1}{\sqrt{2}}\right)^2 = \boxed{\frac{1}{2}}$

$$\frac{(1-\cos^2\alpha)^2+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{(1-\sin^2\alpha)^2+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{1+\cos^4\alpha-2\cos^2\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{1+\sin^4\alpha-2\sin^2\alpha+4\sin^2\alpha}{1+\sin^2\alpha}$$

$$=\frac{1+\cos^4\alpha+2\cos^2\alpha}{1+\cos^2\alpha}-\frac{1+\sin^4\alpha+2\sin^2\alpha}{1+\sin^2\alpha}=\frac{(1+\cos^2\alpha)^2}{1+\cos^2\alpha}-\frac{(1+\sin^2\alpha)^2}{1+\sin^2\alpha}=1+\cos^2\alpha-1-\sin^2\alpha=\boxed{\cos 2\alpha}$$

۶



$$\sin\left(\frac{\pi}{2}+\alpha\right)\cos\left(-\left(\frac{5\pi}{2}+\alpha\right)\right)-\tan\left(-\left(\frac{3\pi}{2}-\alpha\right)\right)$$

$$=\cos\alpha\times\cos\left(\frac{\pi}{2}+\alpha\right)+\tan\left(\frac{3\pi}{2}-\alpha\right)=$$

$$\cos\alpha\times(-\sin\alpha)+\cot\alpha=\frac{3}{5}\left(-\frac{4}{5}\right)+\frac{3}{4}=\frac{-12}{25}+\frac{3}{4}=\frac{-48+75}{100}=\boxed{\frac{27}{100}}$$

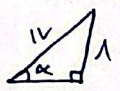
۷

$$3\cos\left(\frac{5\pi}{12}\right)-2\left(\frac{\sqrt{2}}{2}\cos\left(\frac{\pi}{12}\right)-\frac{\sqrt{2}}{2}\sin\left(\frac{\pi}{12}\right)\right)=3\cos\frac{\pi}{3}-2\left(\cos\frac{\pi}{4}\cos\frac{\pi}{12}-\sin\frac{\pi}{4}\sin\frac{\pi}{12}\right)$$

$$=3\times\frac{1}{2}-2\cos\left(\frac{\pi}{4}+\frac{\pi}{12}\right)=\frac{3}{2}-2\cos\frac{\pi}{3}=\frac{3}{2}-2\times\frac{1}{2}=\frac{3}{2}-1=\boxed{\frac{1}{2}}$$

۸

$$\tan\alpha=\frac{2\tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)}=\frac{2\times\frac{1}{4}}{1-\frac{1}{16}}=\frac{\frac{1}{2}}{\frac{15}{16}}=\frac{8}{15}$$



$$x=\sqrt{8^2+15^2}=\sqrt{289}=17\Rightarrow\sin\alpha=\frac{8}{17}\quad\cos\alpha=\frac{15}{17}$$

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\frac{\frac{8(17-15)}{15\times17}}{\frac{-7}{17}}=\boxed{-\frac{16}{105}}$$

۹

$$\frac{\cot\alpha}{\sin\alpha}>0\Rightarrow\frac{\cos\alpha}{\sin^2\alpha}>0\Rightarrow\cos\alpha>0\ (1)\qquad\sin 2\alpha=2\sin\alpha\cos\alpha$$

$$2\sin\alpha<\sin 2\alpha\Rightarrow 2\sin\alpha<2\sin\alpha\cos\alpha\Longrightarrow\sin\alpha<0\ (2)$$

$0<\cos\alpha<1$

↳ اگر $\sin\alpha$ عددی مثبت باشد ضرب شدن آن در عددی بین صفر و یک آن را کوچکتر می‌کند پس باید عددی منفی باشد تا با کم شدن مقدار عددی مقدار حقیقی عدد بیشتر شود.

(1) و $(2)\Rightarrow$ ناحیه چهارم

۱۰