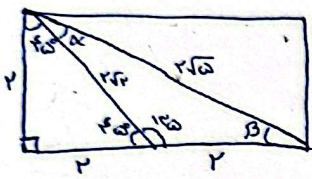


$$S_{ABC} = \frac{1}{2} \Rightarrow \frac{\sqrt{2} \times \sqrt{2} \times \sin \alpha}{2} = \frac{1}{2} \Rightarrow \sin \alpha = \frac{\sqrt{2}}{2} \Rightarrow \alpha_1 = 45^\circ \Rightarrow \frac{\alpha_2}{\alpha_1} = \boxed{2}$$

$$\alpha_2 = 135^\circ$$



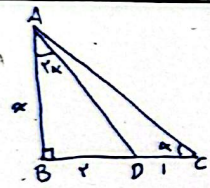
$$\alpha + \beta = 180 - 135 = 45 \quad \sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$$

$$\Rightarrow 2 \sin \alpha + \cos \alpha = \frac{\sqrt{2}}{2} \times \sqrt{2} \quad \textcircled{1} \quad \sin 45^\circ = \sin \alpha \times \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cos \alpha = \frac{2 \sin \alpha + \cos \alpha}{\sqrt{2}}$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \Rightarrow \cos 45^\circ = \cos \alpha \times \frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}} \sin \alpha = \frac{2 \cos \alpha - \sin \alpha}{\sqrt{2}}$$

$$\Rightarrow 2 \cos \alpha - \sin \alpha = \frac{\sqrt{2}}{2} \times \sqrt{2} \quad \textcircled{2}$$

$$\textcircled{1}, \textcircled{2} \Rightarrow 2 \cos \alpha - \sin \alpha = 2 \sin \alpha + \cos \alpha \Rightarrow \cos \alpha = 3 \sin \alpha \Rightarrow \cot \alpha = \frac{\cos \alpha}{\sin \alpha} = \boxed{3}$$



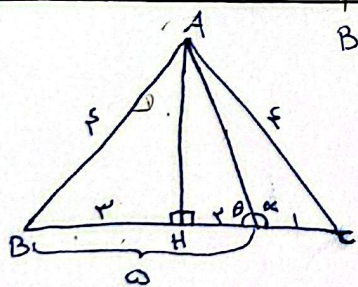
$$AD = \sqrt{a^2 + x^2} \quad \cot 2\alpha = \frac{x}{y} = \frac{\cos 2\alpha}{\sin 2\alpha} = \frac{\cos^2 \alpha - \sin^2 \alpha}{2 \sin \alpha \cos \alpha}$$

$$AC = \sqrt{a^2 + x^2}$$

$$\Rightarrow \frac{x}{y} = \frac{\frac{a}{x^2 + a} - \frac{x^2}{a^2 + a}}{2 \times \frac{x}{\sqrt{a^2 + a}} \times \frac{y}{\sqrt{a^2 + a}}} = \frac{\frac{a - x^2}{a^2 + a}}{\frac{2xy}{a^2 + a}} = \frac{a - x^2}{2xy}$$

$$\Rightarrow 11 - 2x^2 = 9x^2 \Rightarrow 11x^2 = 11 \Rightarrow x^2 = \frac{11}{1} = \frac{9}{1} \Rightarrow x = \frac{3}{1}$$

$$\cot \alpha = \frac{y}{x} = \frac{3}{3} = \boxed{1}$$



در مثلث مستطی ارتفاع و ارتفاع وارد بر پایه (BC) عمود منصف آن است. $BH = CH = 2$

$$AH = \sqrt{5^2 - 2^2} = \sqrt{21}$$

$$\tan \alpha = -\tan \theta = -\frac{\sqrt{21}}{2}$$

$$\sin^2 \alpha + \sin^2 \alpha + \cos^2 \alpha = \frac{4}{3} \Rightarrow \sin^2 \alpha = \frac{1}{3} \Rightarrow \sin \alpha = \pm \frac{1}{\sqrt{3}}$$

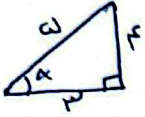


$$\tan^2 \alpha = \left(\pm \frac{1}{\sqrt{3}}\right)^2 = \boxed{\frac{1}{3}}$$

$$\frac{(1-\cos^2\alpha)^{\frac{1}{2}} + \sqrt{\cos^2\alpha}}{1+\cos^2\alpha} - \frac{(1-\sin^2\alpha)^{\frac{1}{2}} + \sqrt{\sin^2\alpha}}{1+\sin^2\alpha} = \frac{1+\cos^2\alpha - \sqrt{\cos^2\alpha} + \sqrt{\cos^2\alpha}}{1+\cos^2\alpha} - \frac{1+\sin^2\alpha - \sqrt{\sin^2\alpha} + \sqrt{\sin^2\alpha}}{1+\sin^2\alpha}$$

$$= \frac{1+\cos^2\alpha + \sqrt{\cos^2\alpha}}{1+\cos^2\alpha} - \frac{1+\sin^2\alpha + \sqrt{\sin^2\alpha}}{1+\sin^2\alpha} = \frac{(1+\cos^2\alpha)^{\frac{1}{2}}}{1+\cos^2\alpha} - \frac{(1+\sin^2\alpha)^{\frac{1}{2}}}{1+\sin^2\alpha} = \sqrt{1+\cos^2\alpha} - \sqrt{1+\sin^2\alpha} = \boxed{\cos^2\alpha}$$

۶



$$\sin\left(\frac{\pi}{4} + \alpha\right) \cos\left(-\left(\frac{\pi}{4} + \alpha\right)\right) - \tan\left(-\left(\frac{\pi}{4} - \alpha\right)\right)$$

$$= \cos\alpha \times \cos\left(\frac{\pi}{4} + \alpha\right) + \tan\left(\frac{\pi}{4} - \alpha\right) =$$

$$\cos\alpha \times (-\sin\alpha) + \cot\alpha = \frac{w}{r} \left(-\frac{p}{w}\right) + \frac{r}{p} = \frac{-wr}{rp} + \frac{r}{p} = \frac{-w + w}{100} = \boxed{\frac{2w}{100}}$$

۷

$$\sqrt{2} \cos\left(\frac{\pi}{12}\right) - \sqrt{2} \left(\frac{\sqrt{2}}{2} \cos\left(\frac{\pi}{12}\right) - \frac{\sqrt{2}}{2} \sin\left(\frac{\pi}{12}\right) \right) = \sqrt{2} \cos\frac{\pi}{12} - \sqrt{2} \left(\cos\frac{\pi}{12} \cos\frac{\pi}{12} - \sin\frac{\pi}{12} \sin\frac{\pi}{12} \right)$$

$$= \sqrt{2} \times \frac{1}{2} - \sqrt{2} \cos\left(\frac{\pi}{12} + \frac{\pi}{12}\right) = \frac{\sqrt{2}}{2} - \sqrt{2} \cos\frac{\pi}{6} = \frac{\sqrt{2}}{2} - \sqrt{2} \times \frac{1}{2} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = \boxed{\frac{1}{2}}$$

۸

$$\tan\alpha = \frac{r \tan\left(\frac{\alpha}{2}\right)}{1 - \tan^2\left(\frac{\alpha}{2}\right)} = \frac{r \times \frac{1}{2}}{1 - \frac{1}{16}} = \frac{\frac{r}{2}}{\frac{15}{16}} = \frac{r}{15}$$

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{r}{15} - \frac{1}{17}}{\frac{1}{17} - \frac{15}{17}} = \frac{\frac{r(17-15)}{15 \times 17}}{\frac{-14}{17}} = \boxed{-\frac{17}{105}}$$

$x = \sqrt{1^2 + 16^2} = \sqrt{257} = 16 \Rightarrow \sin\alpha = \frac{1}{17}$
 $\cos\alpha = \frac{16}{17}$

۹

$$\frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \frac{\cos\alpha}{\sin^2\alpha} > 0 \Rightarrow \cos\alpha > 0 \text{ ①} \quad \sin^2\alpha = 2 \sin\alpha \cos\alpha$$

$$2 \sin\alpha < \sin^2\alpha \Rightarrow 2 \sin\alpha < 2 \sin\alpha \cos\alpha \Rightarrow \sin\alpha < 0 \text{ ②}$$

← اگر $\sin\alpha$ عددی مثبت باشد ضرب شدن آن در عددی بین صفر و یک آن را کوچکتر می کند پس باید عددی منفی باشد تا با کم شدن مقدار عددی مقدار حقیقی عدد بیشتر شود.

① و ② \Rightarrow ناحیه چهارم

۱۰