هلیا غفاری

۱۰/۲۵

(۱)

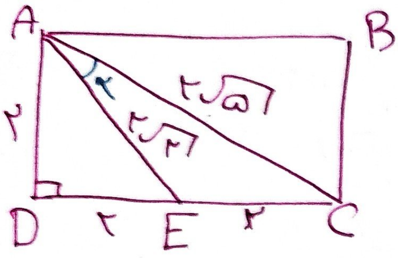
$S_\Delta = \frac{1}{2}ab\sin\alpha \to \frac{9}{2} = 6\times\sqrt{3}\times\frac{1}{2}\times\sin\alpha$

$\Rightarrow \sin\alpha = \frac{9}{2}\times\frac{1}{9}\times\frac{1}{\sqrt{3}}\times 2 = \frac{3}{2\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$ $\begin{cases}\alpha = 60\\ \alpha = 120\end{cases}$

$\frac{\alpha_{max}}{\alpha_{min}} = \frac{120}{60} = 2$

---

(۲)



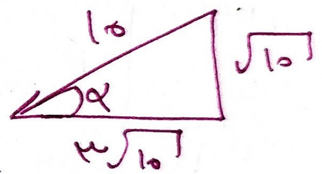
$S_{AEC} = S_{ACD} - S_{ADE}$

$S_{ACD} = \frac{1}{2}\times 4\times 2 = 4$

$S_{ADE} = \frac{1}{2}\times 2\times 2 = 2$ $\Big\}\ S_{AEC} = 2$

$S_{AEC} = \frac{1}{2}\times AC\times AE\times\sin\alpha \xrightarrow[\text{طبق فیثاغورث}]{AE \text{ و } AC} 2 = 2\sqrt{2}\times 2\sqrt{5}\times\frac{1}{2}\times\sin\alpha$

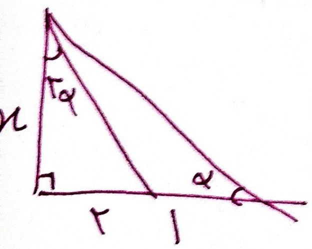
$\sin\alpha = 2\times\frac{1}{2\sqrt{2}}\times\frac{1}{2\sqrt{5}}\times 2 = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$



$\Rightarrow \cos\alpha = \frac{3\sqrt{10}}{10} \Rightarrow \cot\alpha = \frac{3\sqrt{10}}{\sqrt{10}} = 3$

---

(۳)

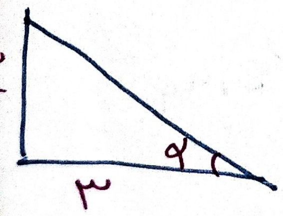


$\tan\alpha = \frac{x}{3}$ و $\tan 2\alpha = \frac{2}{x}$

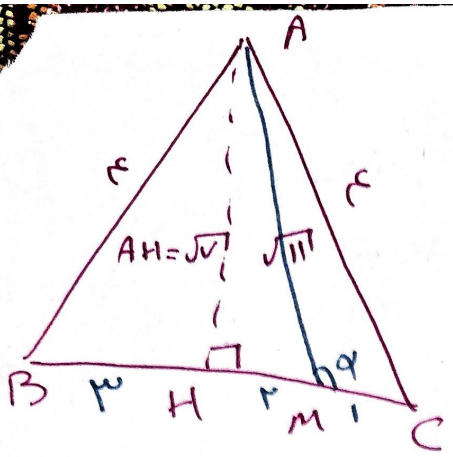
$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \Rightarrow \frac{2}{x} = \frac{\frac{2x}{3}}{1-\frac{x^2}{9}}$

$\Rightarrow \frac{\frac{2x}{3}}{\frac{9-x^2}{9}} = \frac{2}{x} \Rightarrow \frac{6x}{9-x^2}$

$\Rightarrow 18 - 2x^2 = 6x^2 \to 18 = 8x^2 \to 9 = 4x^2 \to x^2 = \frac{9}{4}$ $\xrightarrow[\text{ضلع مثبت است}]{\text{طول}} x = \frac{3}{2}$



$\Rightarrow \cot\alpha = \frac{3}{\frac{3}{2}} = 2$



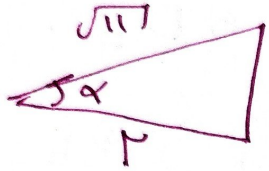
$S_{AMC} = S_{AHC} - S_{AHM}$

$S_{AMH} = \frac{1}{2} \times \sqrt{7} \times 2 = \sqrt{7}$

$S_{AHC} = \frac{1}{2} \times \sqrt{7} \times 3 = \frac{3\sqrt{7}}{2}$

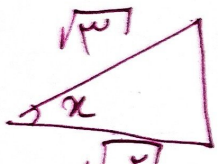
$S_{AMC} = \frac{3\sqrt{7}}{2} - \frac{2\sqrt{7}}{2} = \frac{\sqrt{7}}{2}$

$\Rightarrow \frac{1}{2} \times \sqrt{11} \times 1 \times \sin\alpha = \frac{\sqrt{7}}{2} \Rightarrow \sin\alpha = \frac{\sqrt{7}}{2} \times \frac{2}{\sqrt{11}} = \frac{\sqrt{7}}{\sqrt{11}}$



$\Rightarrow \tan\alpha = \frac{\sqrt{7}}{2}$

$\sin^2 x + \sin^2 x + \cos^2 x = \frac{4}{3} \Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \sin x = \frac{1}{\sqrt{3}}$

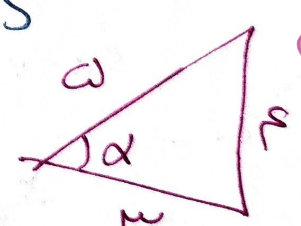


$\tan x = \frac{1}{\sqrt{2}} \Rightarrow \tan^2 x = \frac{1}{2}$

$-\cos\alpha \cdot \sin\alpha + \tan = \frac{-\cos^2 \cdot \sin - \sin}{\cos}$

$= \frac{-\sin(+\cos^2 + 1)}{\cos}$

$\cos^2 = \frac{9}{25} + 1$



$= -\frac{8}{5}\left(\frac{34}{25}\right) = \frac{-8 \times 34}{25 \times 3} = -\frac{136}{75}$

$2\sin\alpha < \sin 2\alpha \Rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \Rightarrow 2\sin\alpha - 2\sin\alpha\cos\alpha < 0$

$\Rightarrow \cos\alpha > 1$

مرفوض

۱۰)

$$\begin{cases} \frac{\cot\alpha}{\sin\alpha} > 0 \rightarrow \frac{\cos}{\sin^2\alpha} > 0 \\ 2\sin\alpha < \sin 2\alpha \rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \rightarrow \begin{cases} \sin^2\alpha > 0 \rightarrow \cos\alpha > 0 \\ \sin\alpha(\cos\alpha - \sin\alpha) > 0 \rightarrow \sin\alpha < 0 \end{cases} \end{cases}$$

$\rightarrow$ ۴ ناصفر

۷) $\sin\left(\frac{9\pi}{2} + \alpha\right) = \cos\alpha \qquad \cos\left(\frac{7\pi}{2} - \alpha\right) = -\sin\alpha$

$\tan\left(\alpha - \frac{3\pi}{2}\right) = -\cot\alpha \qquad \rightarrow \frac{-3}{5} \times \frac{4}{5} + \frac{3}{4} = \frac{27}{100}$