نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۷ کلاس .....

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$$S=\frac{1}{2}\times 4\times\sqrt{3}\times\sin\alpha=4\sqrt{3}\Rightarrow\sin\alpha=\frac{4\sqrt{3}}{4\sqrt{3}}=\frac{\sqrt{3}}{2}\Rightarrow\begin{cases}\alpha=\frac{\pi}{3}\\ \alpha=\frac{2\pi}{3}\end{cases}\Rightarrow\frac{a_{max}}{a_{min}}=\frac{\frac{2\pi}{3}}{\frac{\pi}{3}}=2$$

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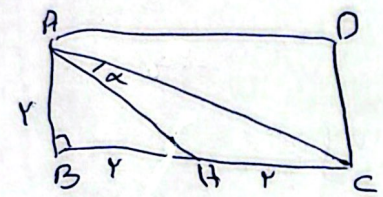

$$\triangle ABC:\tan\beta=\frac{BC}{AB}=\frac{4}{2}=2$$

$$\tan\alpha=\tan(\beta-45)=\frac{\tan\beta-\tan45}{1+\tan\beta\cdot\tan45}=\frac{2-1}{1+2}=\frac{1}{3}$$

$$\cot\alpha=\frac{1}{\tan\alpha}=\frac{1}{\frac{1}{3}}=3$$

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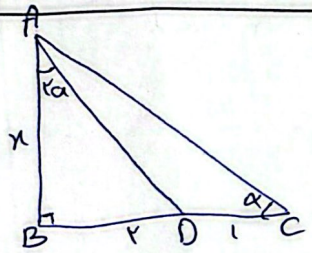

$$\tan\alpha=\frac{x}{2}$$

$$\tan2\alpha=\frac{x}{3x}$$

$$\tan2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}\Rightarrow\frac{x}{3x}=\frac{2\times\frac{x}{2}}{1-\frac{x^2}{4}}\Rightarrow\frac{1}{3x}=\frac{x}{4-x^2}$$

$$\Rightarrow 12-3x^2=4x^2\Rightarrow x^2=\frac{12}{7}\Rightarrow x=\frac{2\sqrt{3}}{\sqrt{7}}\qquad\cot\alpha=\frac{2}{x}=\frac{2}{\frac{2\sqrt{3}}{\sqrt{7}}}=\sqrt{7}$$

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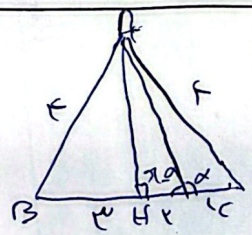

$$AB^2=AH^2+BH^2\Rightarrow 16=AH^2+9\rightarrow AH=\sqrt{7}$$

$$\tan(\pi-\alpha)=\frac{\sqrt{7}}{2}\Rightarrow-\tan\alpha=\frac{\sqrt{7}}{2}\Rightarrow\tan\alpha=-\frac{\sqrt{7}}{2}$$

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$$2\sin^2x+\cos^2x=\frac{4}{3}\xrightarrow{\div\cos^2x}2\tan^2x+1=\frac{4}{3}(1+\tan^2x)\Rightarrow 6\tan^2x+3=4+4\tan^2x$$

$$\Rightarrow 2\tan^2x=1\rightarrow\tan^2x=\frac{1}{2}$$

$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{\sin^2\alpha-4\sin^2\alpha+4}{2-\sin^2\alpha}-\frac{\cos^2\alpha-4\cos^2\alpha+4}{2-\cos^2\alpha}$$

$$=\frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha}-\frac{(2-\cos^2\alpha)^2}{2-\cos^2\alpha}\quad(2-\sin^2\alpha)-(2-\cos^2\alpha)=2-\sin^2\alpha-2+\cos^2\alpha=\cos^2\alpha-\sin^2\alpha=\boxed{\cos^2\alpha}$$

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$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)\tan\left(\alpha-\frac{3\pi}{2}\right)-\sin\left(\frac{\pi}{2}+\alpha\right)\cos\left(\frac{3\pi}{2}-\alpha\right)+\tan\left(\frac{5\pi}{2}-\alpha\right)$$

$$=(\cos\alpha)(-\sin\alpha)+\cot\alpha$$

$$\tan\alpha=\frac{4}{3}\begin{cases}\cos\alpha=-\frac{3}{5}\\ \sin\alpha=-\frac{4}{5}\\ \cot\alpha=\frac{3}{4}\end{cases}\qquad\left(-\frac{3}{5}\right)\left(-\left(-\frac{4}{5}\right)\right)+\frac{3}{4}=\frac{27}{100}=0.27$$

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$$3\cos 6x+\sqrt{2}(\sin x-\cos x)=3\cos 6x+2\sin\left(x-\frac{\pi}{4}\right)\to 3\cos 6\left(\frac{\pi}{12}\right)+2\sin\left(\frac{\pi}{12}-\frac{\pi}{4}\right)$$

$$=3\cos\frac{\pi}{2}+2\sin\left(-\frac{\pi}{6}\right)=3(0)+2\left(-\frac{1}{2}\right)=\boxed{-1}$$

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$$\sin\alpha=\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=\frac{2\left(\frac{1}{3}\right)}{1+\left(\frac{1}{3}\right)^2}=\frac{6}{10}\qquad\cos\alpha=\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=\frac{1-\left(\frac{1}{3}\right)^2}{1+\left(\frac{1}{3}\right)^2}=\frac{8}{10}$$

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\sin\alpha-\sin\alpha\cos\alpha}{\sin\alpha\cos\alpha-\cos^2\alpha}=\frac{\frac{6}{10}-\frac{6}{10}\times\frac{8}{10}}{\frac{6}{10}\times\frac{8}{10}-\left(\frac{8}{10}\right)^2}=\boxed{\frac{14}{16}}$$

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$$2\sin\alpha<2\sin\alpha\cos\alpha\quad\to\quad\sin\alpha(\underbrace{1-\cos\alpha}_{(+)})<0\quad\to\quad\sin\alpha=(-)$$

$$\to\cot\alpha=(-)\quad\to\quad\text{ربع دوم}$$

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