

$\sqrt{5} < 4 < 2$ $s = 4, d$ α بغير حساب = 120° α بغير حساب = 90°

$$\frac{s}{\sin \alpha} = \frac{\sqrt{5} \times 4 \times \sin \alpha}{1} = 4, d \rightarrow 4 \sqrt{5} \times \sin \alpha = 4, d \rightarrow \sin \alpha = \frac{4, d}{4 \sqrt{5}} = \frac{\sqrt{5}}{5} \quad (1)$$

$\Rightarrow \alpha = 45^\circ$ $\angle \alpha = 120^\circ$ $\frac{120}{4} = 30$

$AD^2 + DP^2 = AP^2 \rightarrow \sqrt{4+4} = 2\sqrt{2} = AP$

$AC = \sqrt{4+16} = 2\sqrt{5}$

① $S_{ADC} = 2 \times 2 \times \frac{1}{2} = 2$

② $S_{ADC} = \frac{2\sqrt{2} \times 2\sqrt{5} \times \sin \alpha}{2} = 2 \rightarrow 2\sqrt{10} \times \sin \alpha = 4 \rightarrow \sin \alpha = \frac{1}{\sqrt{10}}$

$1 + \cot^2 \alpha = \frac{1}{\sin^2 \alpha} \rightarrow \cot^2 \alpha = 10 - 1 = 9 \rightarrow \cot \alpha = \pm 3 \checkmark$

$\Delta ABC = \tan^2 \alpha = \frac{4}{9}$
 $\Delta ABC = \tan \alpha = \frac{2}{3}$ } $\Rightarrow \frac{\frac{4n}{9-n^2}}{9} = \frac{4}{9} \rightarrow \frac{4n}{9-n^2} = \frac{4}{9} \rightarrow 4n^2 = 18 - 4n^2 \rightarrow 8n^2 = 18 \rightarrow n^2 = \frac{9}{4} \rightarrow n = \frac{3}{2}$

$\tan \alpha = \frac{2}{3} = \frac{1}{1.5} \rightarrow \cot \alpha = 1.5$


$\alpha + \beta = 180^\circ \rightarrow \tan \alpha = -\tan \beta \rightarrow \tan \alpha = \frac{-5\sqrt{2}}{1}$

$BH = HC = 2$
 $HD = 2 - 1 = 1$ } $\rightarrow AD = \sqrt{1+4} = \sqrt{5} \rightarrow \tan \beta = \frac{5\sqrt{2}}{1}$

$AH^2 + BH^2 = AB^2 \rightarrow AH^2 = 17 - 4 = 13 \rightarrow AH = \sqrt{13}$

$\tan = ?$ $2 \sin^2 \alpha + \cos^2 \alpha = \frac{5}{4} \rightarrow \sin^2 \alpha + \sin^2 \alpha + \cos^2 \alpha = \frac{5}{4} \rightarrow \sin^2 \alpha = \frac{1}{4}$

$\rightarrow \sin \alpha = \pm \frac{1}{2}$

 $\rightarrow \tan \alpha = \pm \frac{1}{1} \rightarrow \tan \alpha = \pm 1$

(2)

$$\frac{\sin \alpha + \cos \alpha}{1 + \cos^2 \alpha} - \frac{\cos \alpha + \sin \alpha}{1 + \sin^2 \alpha} = \frac{\sin \alpha + \cos \alpha (1 - \sin^2 \alpha)}{1 + 1 - \sin^2 \alpha} = \frac{\sin \alpha + \cos \alpha - \cos \alpha \sin^2 \alpha}{2 - \sin^2 \alpha} = \frac{(\sin \alpha - \cos \alpha)(1 + \cos \alpha)}{-(\sin^2 \alpha - 1)}$$

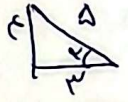
$$\text{کران} = (2 - \sin^2 \alpha) \quad / \quad \text{کران} = \frac{\cos^2 \alpha + 1 - \cos^2 \alpha}{1 + 1 - \cos^2 \alpha} = \frac{\cos^2 \alpha + 1 - \cos^2 \alpha}{2 - \cos^2 \alpha} = \frac{(\cos^2 \alpha - 1)^2}{-(\cos^2 \alpha - 1)}$$

$$\text{کران} = (2 - \cos^2 \alpha) \quad \Rightarrow (2 - \sin^2 \alpha)(2 - \cos^2 \alpha) = -\sin^2 \alpha + \cos^2 \alpha \Rightarrow \frac{2 \cos^2 \alpha}{2} = \cos^2 \alpha$$

$\tan \alpha = \frac{1}{\sqrt{3}} \quad \alpha \rightarrow 30^\circ$

$\sin(\frac{\pi}{4} + \alpha) = (\frac{\sqrt{2}}{2} \cos \alpha - \frac{\sqrt{2}}{2} \sin \alpha) = (\cos \alpha) \times (-\sin \alpha) - (\sin \alpha)$

$\rightarrow -\cos \alpha \sin \alpha + \cos^2 \alpha = -\frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{-1}{3} + \frac{1}{\sqrt{3}} = \frac{-1 + \sqrt{3}}{3}$



$\alpha = \frac{\pi}{12} \quad (\sqrt{2} \cos \alpha + \sqrt{2} \sin \alpha - \sqrt{2} \cos \alpha) = \sqrt{2} \cos \alpha \times \frac{\pi}{12} + \sqrt{2} (\sin \alpha - \cos \alpha)$

$= \sqrt{2} \cos \frac{\pi}{12} + \sqrt{2} (\sin \alpha - \cos \alpha) \xrightarrow{\text{①}} \sqrt{2} \times \frac{1}{2} + \sqrt{2} (\sin \frac{\pi}{4} - \cos \frac{\pi}{4}) = \frac{\sqrt{2}}{2} + \sqrt{2} (\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}) = \frac{\sqrt{2}}{2}$

① $\sin \alpha - \cos \alpha = \sqrt{2} \sin(\alpha - \frac{\pi}{4})$

$\tan(\frac{\alpha}{2}) = \frac{1}{\sqrt{3}}$

$\frac{\tan \alpha - \sin \alpha}{\sin \alpha - \cos \alpha} = \frac{\frac{\tan(\frac{\alpha}{2})}{1 - \tan^2(\frac{\alpha}{2})} - \frac{\tan(\frac{\alpha}{2})}{1 + \tan^2(\frac{\alpha}{2})}}{\frac{\tan(\frac{\alpha}{2})}{1 + \tan^2(\frac{\alpha}{2})} - \frac{1 - \tan^2(\frac{\alpha}{2})}{1 + \tan^2(\frac{\alpha}{2})}} = \frac{\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}} - \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}}} = \frac{\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}} - \frac{2/3}{4/3}} = \frac{\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}} - \frac{1}{2}} = \frac{1}{\sqrt{3}} - \frac{1}{2} = \frac{2 - \sqrt{3}}{2\sqrt{3}}$

$\sqrt{2} \sin \alpha < \sin \alpha \rightarrow \sqrt{2} \sin \alpha < \sqrt{2} \sin \alpha \cos \alpha \rightarrow \sqrt{2} \sin \alpha - \sqrt{2} \sin \alpha \cos \alpha < 0$

$\rightarrow \sqrt{2} \sin \alpha (1 - \cos \alpha) < 0 \rightarrow \sin \alpha < 0$ ①

$\frac{\cos \alpha}{\sin \alpha} > 0 \rightarrow \frac{\cos \alpha}{\sin^2 \alpha} > 0 \rightarrow \cos \alpha > 0$ ②

①, ② = ربع اول (Q1)