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سوال ① =>

$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$, $\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$

$\frac{\cos\alpha}{\sin\alpha} \cdot \frac{\cos\alpha}{|\sin\alpha|} \longrightarrow \sin\alpha > 0$ $\qquad$ $\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} \cdot \frac{1-\sin\alpha}{|\cos\alpha|} \longrightarrow \cos\alpha > 0$

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α در ناحیه اول قرار دارد

سوال ② =>

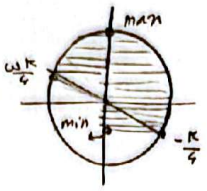
$-\frac{\pi}{12} < x < \frac{3\pi}{12}$, $\sin 2x = \frac{m-1}{4}$

$-\frac{\pi}{6} < 2x < \frac{3\pi}{6} \longrightarrow -\frac{1}{2} < \sin 2x \leq 1$

$-\frac{1}{2} < \frac{m-1}{4} \leq 1$

$-2 < m-1 \leq 4$

$-1 < m \leq 5$



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سوال ③ =>

$\tan x + \cot x = -3$, $3\pi < x < 4\pi$, $\frac{1}{\sin^3 x + \cos^3 x} = ?$

$\tan x + \cot x = \frac{1}{\cos x \sin x} \cdot -3 \longrightarrow \cos x \sin x = -\frac{1}{3}$

$3\pi < x < 4\pi \xrightarrow{\div 4} \frac{3\pi}{4} < x < \pi \longrightarrow \underbrace{\cos x}_{\ominus} < \underbrace{\sin x}_{\oplus}$, $|\cos x| > |\sin x|$

$\sin x + \cos x = -\sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x} = -\sqrt{1 - \frac{2}{3}} = -\frac{1}{\sqrt{3}}$

$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)} = \frac{1}{(-\frac{1}{\sqrt{3}})(1 - (-\frac{1}{3}))} = \frac{1}{-\frac{4}{3\sqrt{3}}} = \frac{-3\sqrt{3}}{4}$

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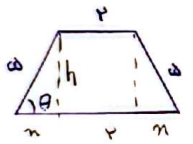
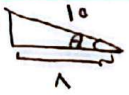
سوال ④ =>



$\cos\theta = 0/6$ ⟵ 

$\cos\theta = \frac{x}{5} = \frac{6}{10} \longrightarrow x = 3$

$\sin\theta = \frac{h}{5} = \frac{8}{10} \longrightarrow h = 4$

$S = \frac{(2+8) \times 4}{2} = 20$

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سوال ⑤ =>

$\tan(2\pi\omega)\tan(-14\omega^\circ) - \sin(109\omega^\circ)\cos(270\omega^\circ) = k\cos^2 19\omega^\circ$ $\qquad$ $k = ?$

$= \tan(\frac{3\pi}{2} + 19\omega) \times (-\tan(\pi - 19\omega)) - \sin(4\pi + 19\omega)\cos(\frac{3\pi}{2} - 19\omega)$

$= -\cot 19\omega \times \tan 19\omega - \sin 19\omega \times (-\sin 19\omega) = -1 + \sin^2 19\omega = -\cos^2 19\omega$

$k = -1$

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سوال ⑥ =>

$$A=\frac{\sqrt{3}\cos(210^\circ)\sin(\frac{3\pi}{2}-2\pi)-\sqrt{2}\sin(\frac{5\pi}{2}-2\pi)\cos(\frac{\pi}{2}-2\pi)}{\cos(270^\circ)} = ?$$

$$= \frac{\sqrt{3}\times(-\frac{\sqrt{3}}{2})\times\sin(\frac{3\pi}{2}-2\pi)-\sqrt{2}(\frac{\sqrt{2}}{2})\times\cos(\frac{\pi}{2}-2\pi)}{\cos 270} \cdot \frac{\frac{-3}{2}\times(-\cos 270)-\frac{1}{2}\times(-\cos 270)}{\cos 270} = \frac{\frac{3}{2}\cos^2 270+\cos^2 270}{\cos 270}$$

$$= \frac{\frac{5}{2}\cos 270}{\cos 270}\cdot\frac{5}{2}$$

سوال ⑦ =>

$$f(x)=16\cos^2(15x)\cos^2(45x)\cos^2(135x)\cos^2(315x), \quad f(\frac{\pi}{315})=?$$

$$f(x)=16\times\frac{(1+\cos 30x)}{2}\times\frac{(1+\cos 90x)}{2}\times\frac{(1+\cos 270x)}{2}\times\frac{(1+\cos 630x)}{2}$$

$$f(x)=(1+\cos 30x)(1+\cos 90x)(1+\cos 270x)(1+\cos 630x)$$

$$f(x)=(1+\frac{\sqrt{3}}{2})(1+\frac{1}{2})(1+\frac{-1}{2})(1-\frac{1}{2})=\frac{9+4\sqrt{3}}{16}$$

سوال ⑧ =>

$$\frac{1-\sin x}{1+\sin x}=4 \text{ و } x \text{ در ربع سوم} \text{ انتهای کمان} \quad , \quad \tan\frac{x}{2}=?$$

$$\hookrightarrow 1-\sin x=4+4\sin x$$

$$\sin x=-\frac{3}{5} \rightarrow \sin x=\frac{-3}{5} \rightarrow \cos x=\frac{-4}{5}$$

$$\tan x=\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \longrightarrow \frac{3}{4}=\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \rightarrow 3-3\tan^2\frac{x}{2}=8\tan\frac{x}{2} \rightarrow 3\tan^2\frac{x}{2}+8\tan\frac{x}{2}-3=0$$

$$\tan^2\frac{x}{2}+8\tan^2\frac{x}{2}-9=0$$

$$\tan\frac{x}{2}=1,-9 \longleftarrow \tan\frac{x}{2}\begin{cases}\frac{1}{2}\\ -3\end{cases}$$

سوال ⑨ =>

$$\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=k\cot\frac{\theta}{2} \quad , \quad k=?$$

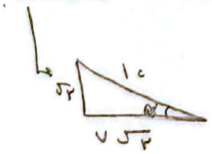
$$\star \quad \frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}=\tan\frac{\alpha}{2} \longrightarrow \text{کسرها معکوس} = \cot\frac{\alpha}{2}$$

$$\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\cot\frac{\theta}{2}+\cot\frac{\theta}{2}=2\cot\frac{\theta}{2} \longrightarrow \boxed{k=2}$$

سوال ⑩ =>

$$\sin\alpha=\frac{\sqrt{2}}{10} \text{ و } \alpha \text{ انتهای کمان در ربع دوم} \quad , \quad \cos(\frac{11\pi}{4}+\alpha)=?$$

$\sin>0 \quad \cos<0$



$$\cos\alpha=\frac{\sqrt{7}}{10}$$

$$\cos(\frac{11\pi}{4}+\alpha)=\cos\frac{11\pi}{4}\cos\alpha-\sin\frac{11\pi}{4}\sin\alpha=(-\frac{\sqrt{2}}{2}\times\frac{\sqrt{7}}{10})-(\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10})$$

$$=\frac{\sqrt{7}}{10}-\frac{1}{10}=\frac{6}{10}=\frac{3}{5}$$