نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۸ ... کلاس .....

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$\frac{1}{\sqrt{\cos^2\alpha}} = \frac{1}{|\cos\alpha|}$, $\frac{1}{\cot\alpha} = \frac{\sin\alpha}{\cos\alpha}$ $\Big\}$ $\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha}$ $\begin{cases} \cos\alpha > 0 \to \frac{1-\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{\cos\alpha} & \text{قبول} \\ \cos\alpha < 0 \to \frac{-1-\sin\alpha}{\cos\alpha} = \frac{\sin\alpha - 1}{\cos\alpha} & \text{غ ق} \end{cases}$ $\Rightarrow \cos\alpha > 0$

$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \sin\alpha = |\sin\alpha| \Rightarrow \sin\alpha > 0 \to$ ناحیه اول سلطانی

۲

$-\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{4} < 2x < \frac{5\pi}{4} \to \sin(\frac{5\pi}{4}) = \frac{1}{2}$, $\sin(-\frac{\pi}{4}) = -\sin(\frac{\pi}{4}) = -\frac{1}{2}$, $\sin(\frac{\pi}{2}) = 1$

$\Rightarrow -\frac{1}{2} < \sin 2x \leq 1 \to -\frac{1}{2} < \frac{m-1}{4} \leq 1 \to -2 < m-1 \leq 4 \to -1 < m \leq 5$

۳

$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\frac{1}{2}\sin 2x} = \frac{2}{\sin 2x} = -4 \to \sin 2x = -\frac{2}{4}$

$\sin^3 x + \cos^3 x = (\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x) = (\sin x + \cos x)(1 - (\frac{1}{2})\sin 2x)$

$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x \cos x = 1 + \sin 2x = 1 - \frac{2}{4} = \frac{1}{4}$

$\to \frac{\pi}{4} < x < \pi \Rightarrow |\cos x| > |\sin x|$, $\cos x < 0 \to \sin x + \cos x < 0 \to \sin x + \cos x = \frac{-1}{\sqrt{2}}$

$\to \sin^3 x + \cos^3 x = (\frac{-1}{\sqrt{2}})(1 - \frac{1}{2} \times (\frac{-2}{4}) = (\frac{-1}{\sqrt{2}})(\frac{3}{4}) = \frac{-3}{4\sqrt{2}} \to \frac{1}{\sin^3 x + \cos^3 x} = \frac{-4\sqrt{2}}{3}$

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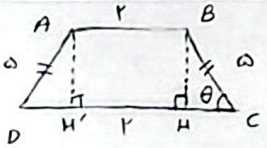

$\triangle BCH$: $\cos\theta = \frac{CH}{BC} = \frac{CH}{5} = \frac{4}{10} \to CH = 4$

$\triangle BCH \cong \triangle ADH' \Rightarrow DH' = CH = 4$

$ABHH' \to$ مستطیل $\Rightarrow HH' = 2 \to CD = 2 + 4 + 4 = 10$

$\triangle BCH$: $BH^2 = BC^2 - CH^2 = 20 - 9 = 14 \to BH = 4$

$S = \frac{(10+2) \times 4}{2} = 24$

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$\tan(24\omega) = \tan(\frac{24\pi}{2} + \omega) = -\cot(\omega)$

$\tan(114\omega) = -\tan(114\omega) = -\tan(\pi - 3\omega) = \tan(3\omega)$

$\sin(109\omega) = \sin(\omega)$

$\cos(23\omega) = \cos(\frac{12\pi}{2} - \omega) = -\sin(\omega)$

$\to -\cot(\omega) \times \tan\omega - (-\sin^2(\omega)) = \sin^2\omega - 1 = -\cos^2(\omega) = k\cos^2\omega \to k = -1$

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$\cos(210) = -\frac{\sqrt{3}}{2}$

$\sin(\frac{14\pi}{3}) = \sin(\frac{12\pi}{3} - 2\pi) = -\cos(2\pi)$

$\sin(1350) = \frac{\sqrt{2}}{2}$

$\cos(1035) = \cos(\pi - 2\pi) = -\cos(2\pi)$

$\frac{\sqrt{3}}{2}(\sqrt{2})(-\cos(2\pi)) + \sqrt{2}(\frac{\sqrt{2}}{2})(\cos(2\pi)) = \frac{\sqrt{6}}{2}\cos 2\pi + \cos 2\pi = \frac{\sqrt{6}+2}{2}\cos 2\pi$ برابر $\frac{\sqrt{6}+2}{2}$

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عبارت را در $\sin^2(\frac{\pi}{12})$ ضرب و تقسیم می‌کنیم.

$\cos^2(12x)\sin^2(12x) = \frac{1}{4}\sin^2(24x)$

$\frac{1}{16}\sin^2(24x)\cos^2(48x) = \frac{1}{16}\sin^2(48x) \rightarrow \frac{1}{16}\sin^2(48x)\cos^2(48x) = \frac{1}{16\times 4}\sin^2(96x)$

$\frac{1}{16\times 4}\sin^2(96x)\cos^2(96x) = \frac{1}{16\times 16}\sin^2(192x) \rightarrow \frac{\frac{1}{16\times 16}\sin^2(192x)\times 16}{\sin^2(12x)} =$

$(\frac{\sin(48x)}{4\sin(12x)})^2 \rightarrow \sin(48x) = \sin(\frac{4\pi}{3}) = \frac{-\sqrt{3}}{2}$, $\sin(12x) = \sin\frac{\pi}{12} = \sin 15 = \sin(60-45)$

$= \frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\times\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4} \rightarrow (\frac{-\frac{\sqrt{3}}{2}}{\sqrt{6}-\sqrt{2}})^2 = \frac{\frac{3}{4}}{8-4\sqrt{12}} = \frac{3}{16(2-\sqrt{3})} = \frac{3(2+\sqrt{3})}{16} = \frac{6+3\sqrt{3}}{16}$

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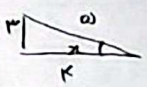
$\frac{1-\sin x}{1+\sin x} = 4 \rightarrow 1-\sin x = 4+4\sin x \rightarrow -3 = 5\sin x \rightarrow \sin x = \frac{-3}{5}$



$\rightarrow \tan x = \frac{3}{4} \rightarrow \tan\frac{x}{2} = a \rightarrow \tan x = \frac{2a}{1-a^2} = \frac{3}{4} \Rightarrow 8a = 3-3a^2$

$3a^2 + 8a - 3 = 0 \rightarrow a^2 + 8a - 9 = (a+9)(a-1) \rightarrow a = \frac{-9}{3} = -3,\ a = \frac{1}{3}$

$\tan\frac{x}{2} \leftarrow$ مقدار صحیح -3

۹

$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} \rightarrow \frac{\sin^2\theta + 1 - \cos^2\theta}{\sin\theta(1-\cos\theta)} = \frac{2\sin^2\theta}{\sin\theta(1-\cos\theta)} = \frac{2\sin\theta}{(1-\cos\theta)} = \frac{2\sin\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}$

$= \frac{2\sin\theta(1+\cos\theta)}{1-\cos^2\theta} = \frac{2\sin\theta(1+\cos\theta)}{\sin^2\theta} = \frac{2(1+\cos\theta)}{\sin\theta} = \frac{4\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = 2\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} =$

$= 2\cot\frac{\theta}{2} = k\cot\frac{\theta}{2} \rightarrow k = 2$

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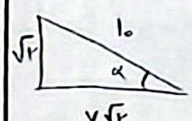

$\cos(\frac{11\pi}{4} + \alpha) = \cos(\frac{11\pi}{4})\cos\alpha - \sin(\frac{11\pi}{4})\sin(\alpha) = -\frac{\sqrt{2}}{2}\times(-\frac{7\sqrt{2}}{10}) - \frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}$

$= \frac{14}{20} - \frac{2}{20} = \frac{12}{20} = \frac{4}{10} = \frac{2}{5}$

$\frac{11\pi}{4} = \frac{8\pi+3\pi}{4} = 2\pi + \frac{3\pi}{4} \rightarrow \cos(2\pi + \frac{3\pi}{4}) = \cos(\frac{3\pi}{4}) = \frac{-\sqrt{2}}{2}$

$\sin(2\pi + \frac{3\pi}{4}) = \sin(\frac{3\pi}{4}) = \frac{\sqrt{2}}{2}$